Universiteit Utrecht

A Gentle Introduction to Bayesian Statistics

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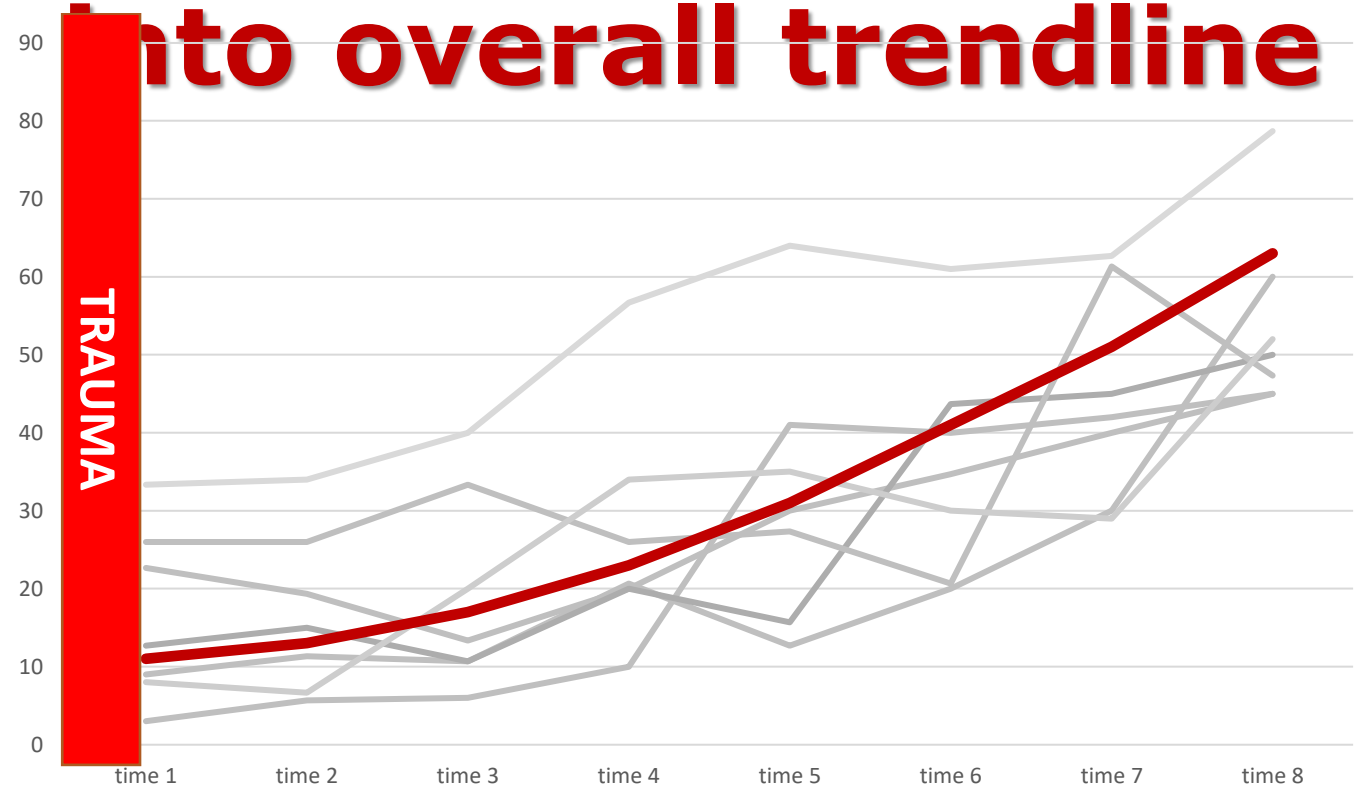
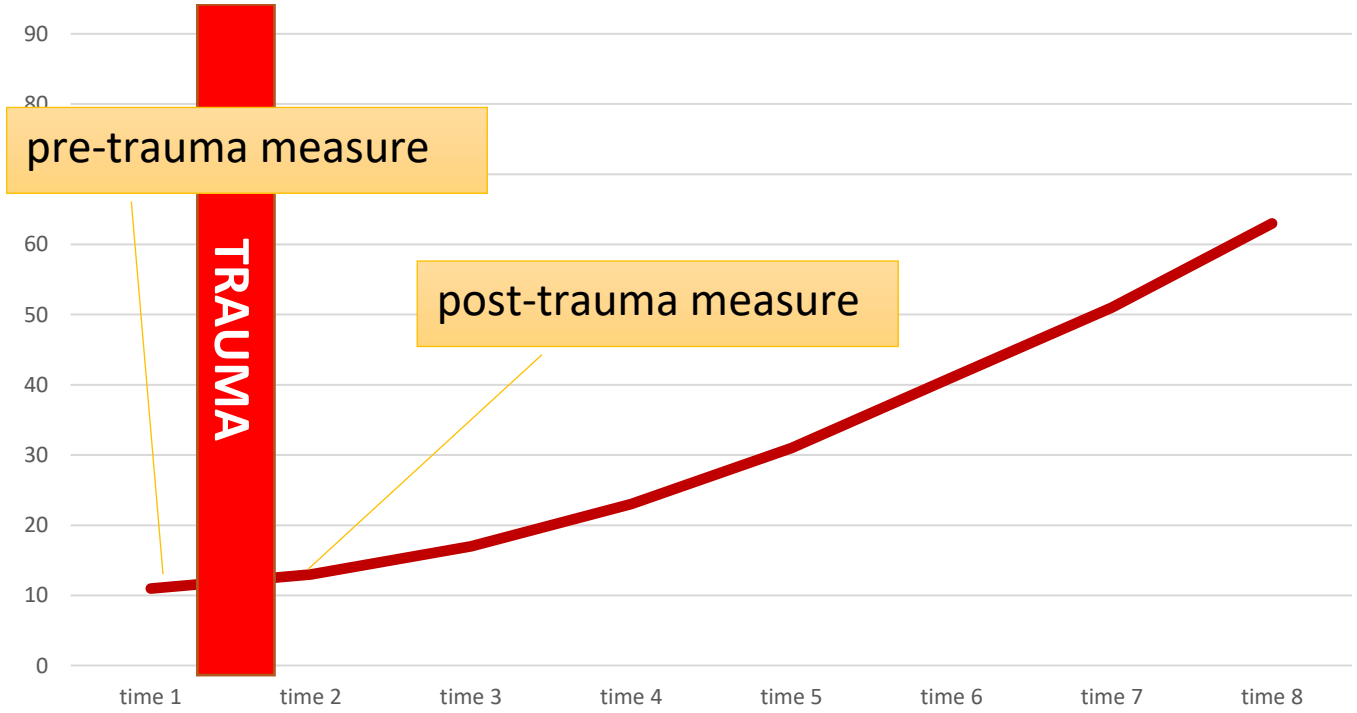
An Example

Individual trajectories summarized into overall trendline

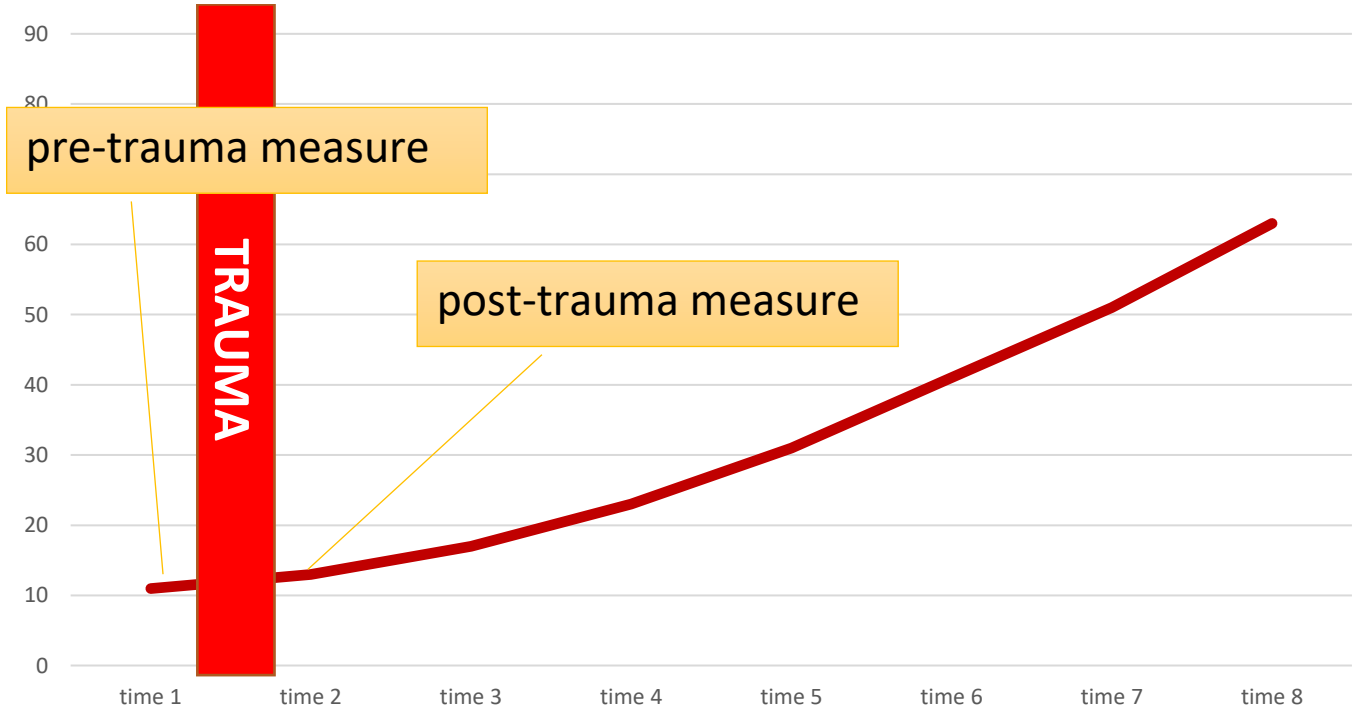


Assumption: Measurement Invariance



Lommen, M. Van de Schoot, R. and Engelhard, I. (2014). The experience of traumatic events **disrupts** the measurement invariance of a posttraumatic stress scale. Frontiers in Psychology, 5:1304

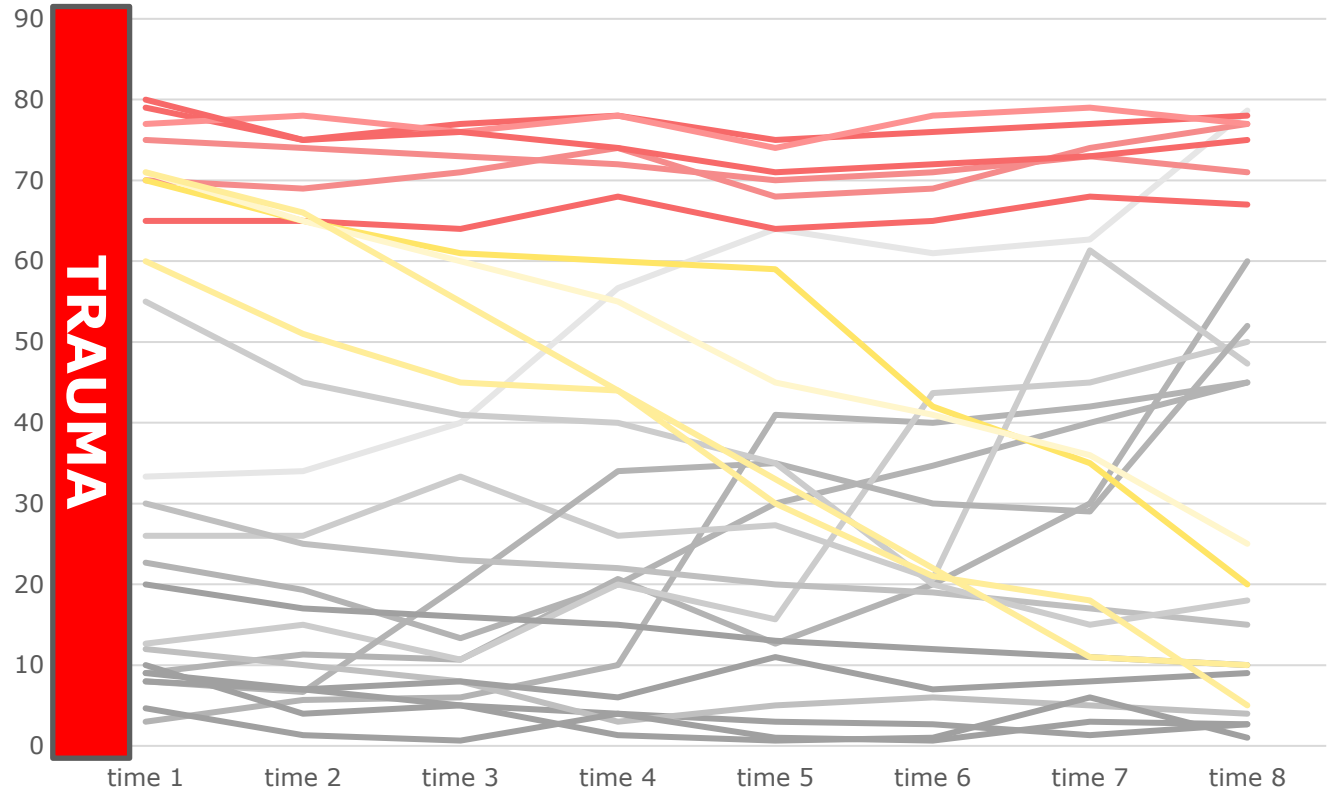
Assumption: Measurement Invariance



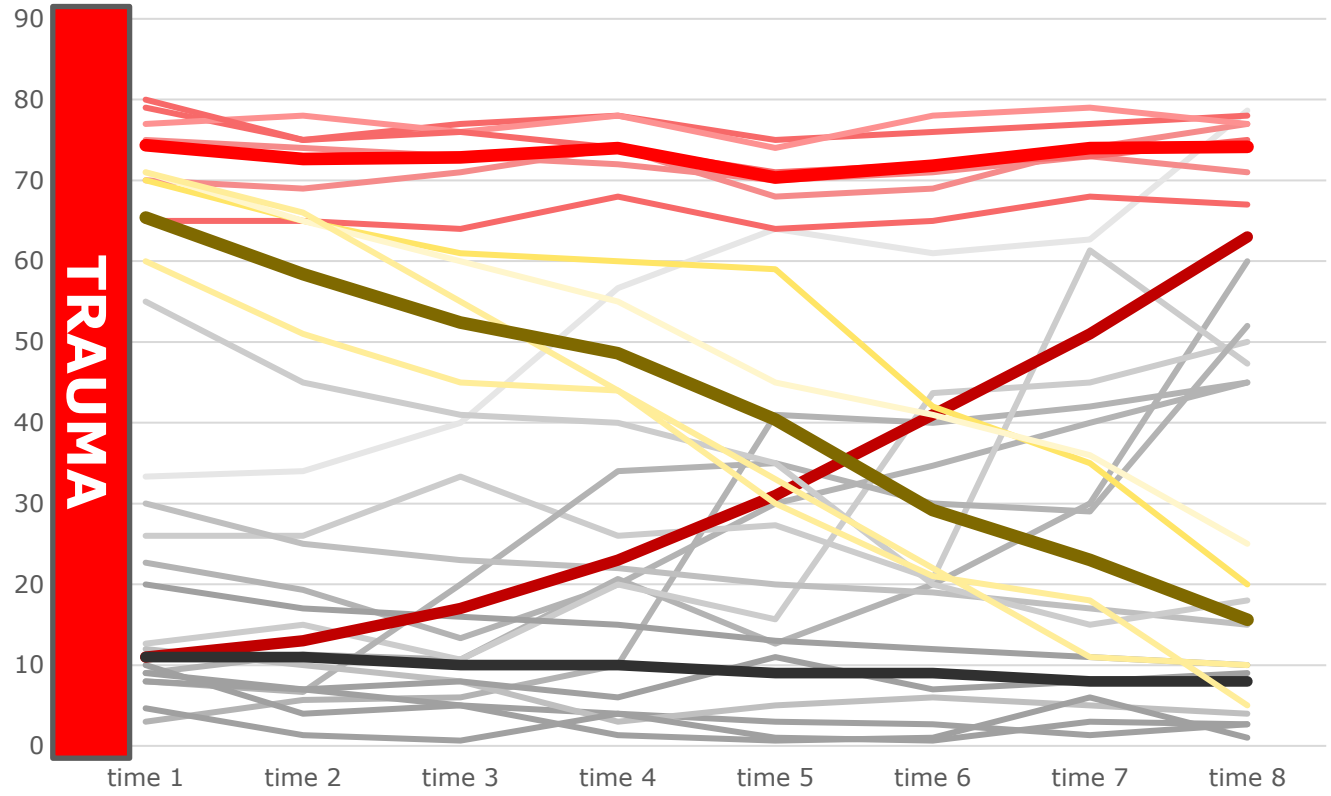
More on measurement invariance in the special issue:

van de Schoot, R., Schmidt, P., De Beuckelaer, A., eds. (2015). Measurement Invariance. Lausanne: Frontiers in Psychology.
http://journal.frontiersin.org/researchtopic/1695

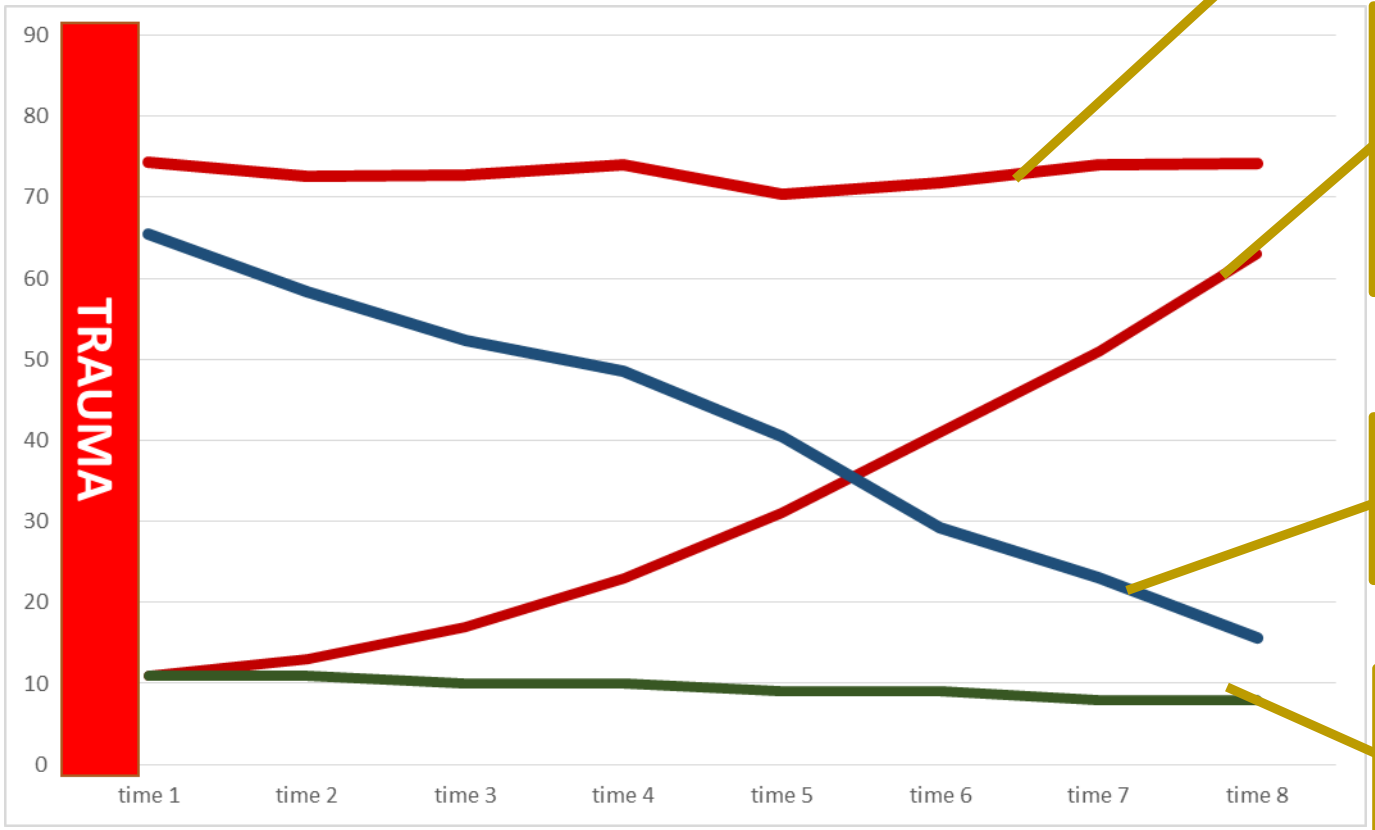
Individual trajectories



Latent subgroups



Latent subgroups



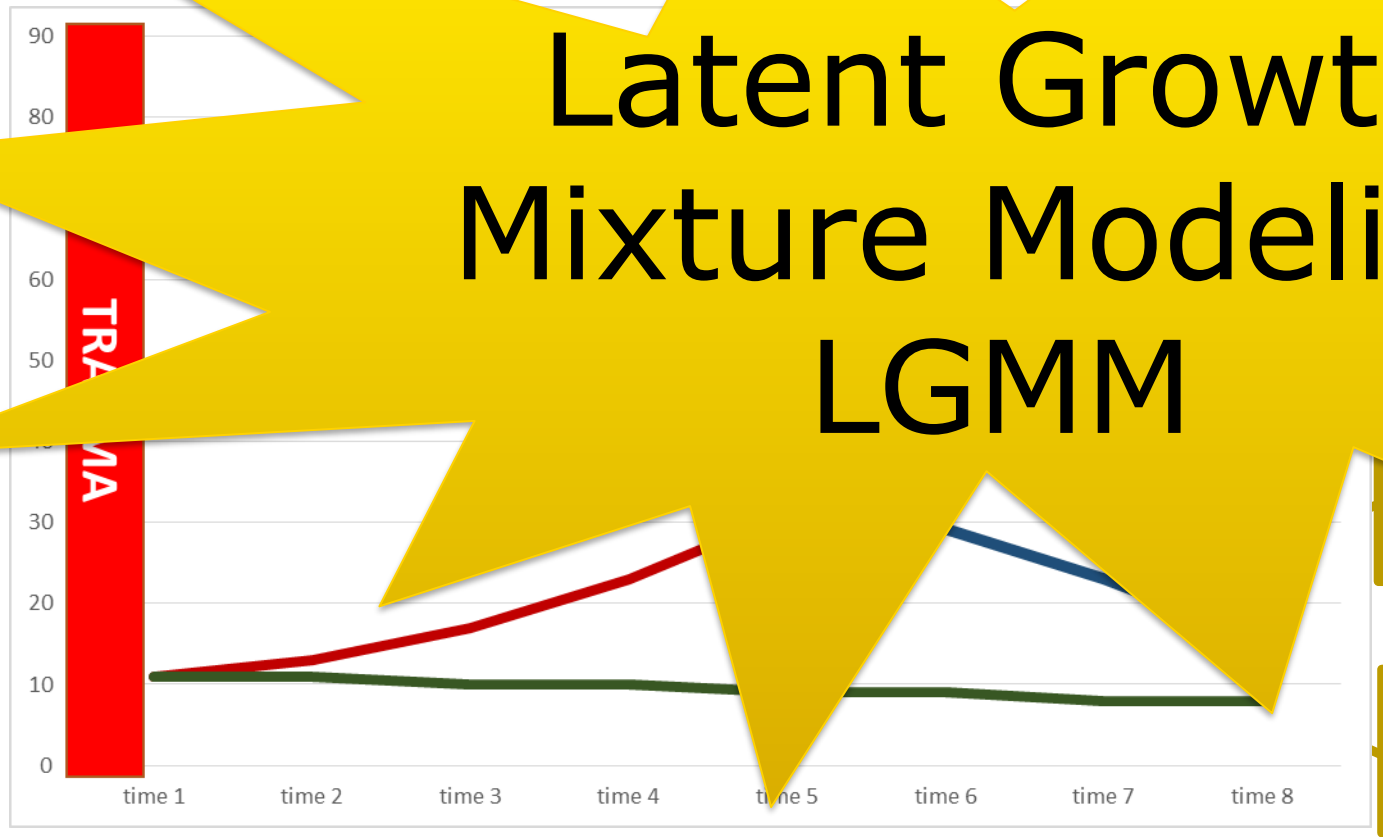
Chronic

Delayed onset / worsening

Recovery

Resilient

Latent subgroups



Chronic

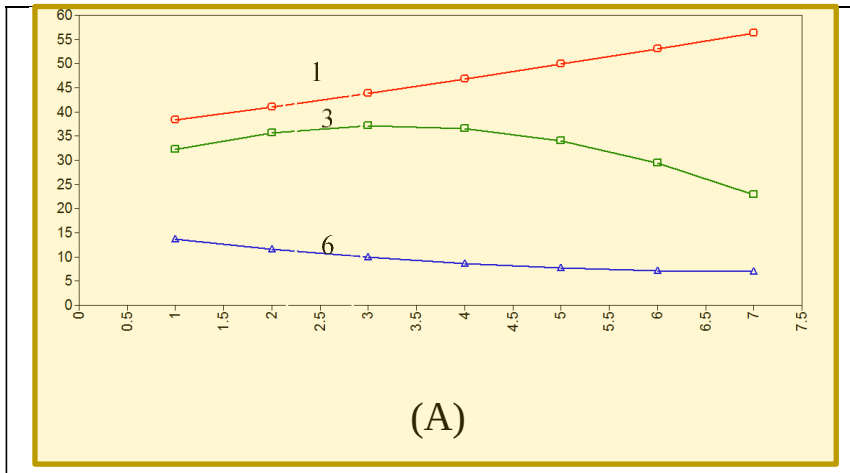
Resilient

Latent Growth Mixture Modeling LGMM

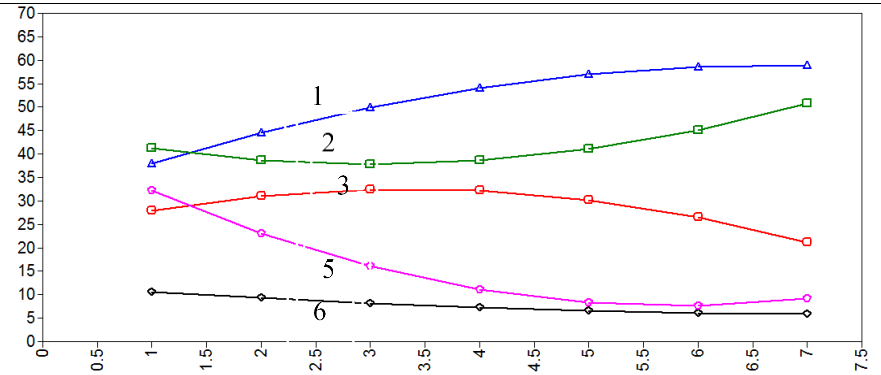
Default LGMM

- Starts with estimating a single cluster
- Next, several additional models are estimated with an increasing number of clusters.
- For each of these LGMM-models, the software is allowed to estimate all parameters without any restrictions.
- A final model is chosen based on model comparison tools, for example using the BIC

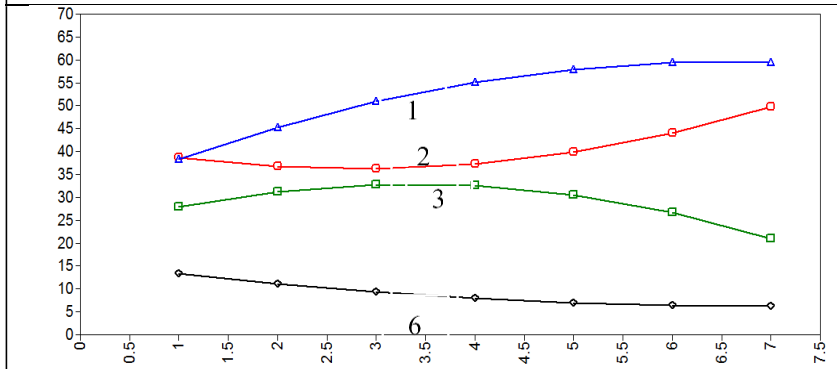
Empirical Example



(A)



(C)



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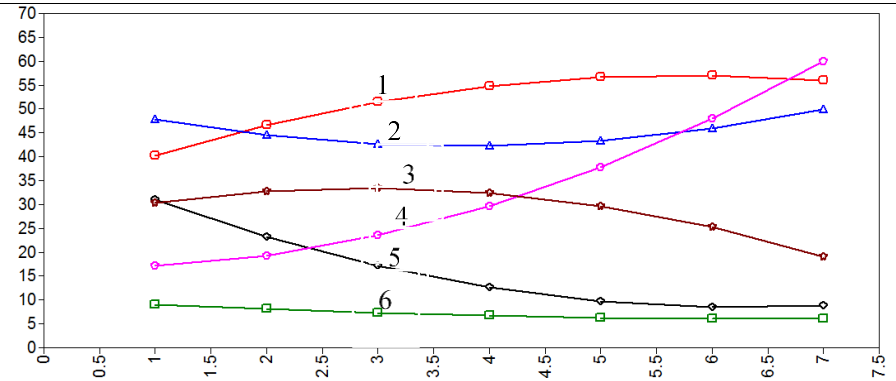
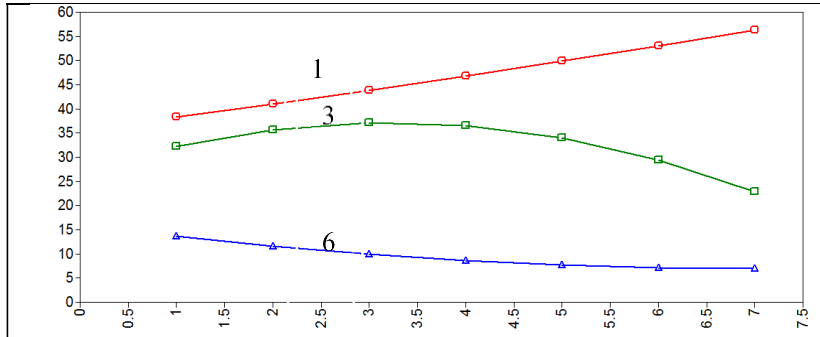


(D)

Empirical Example



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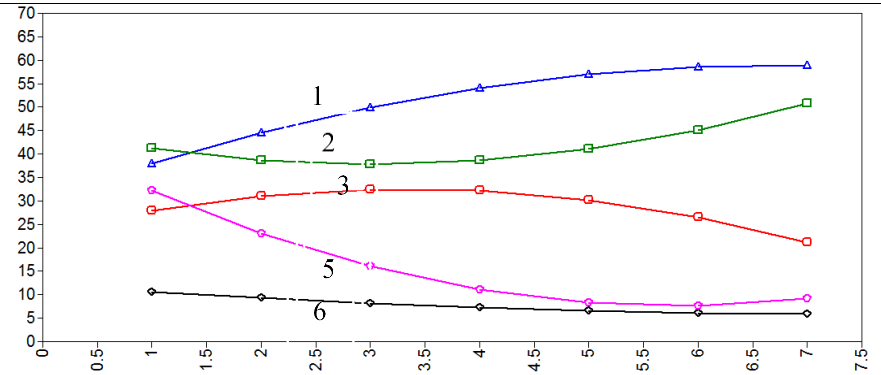

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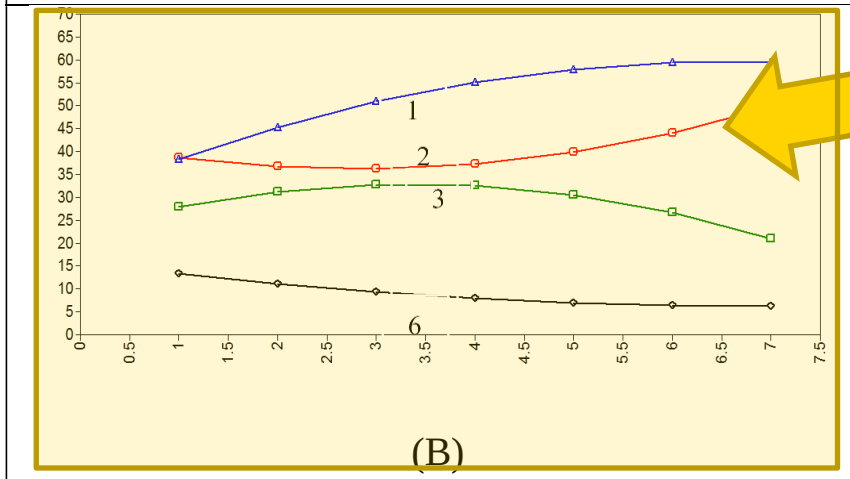

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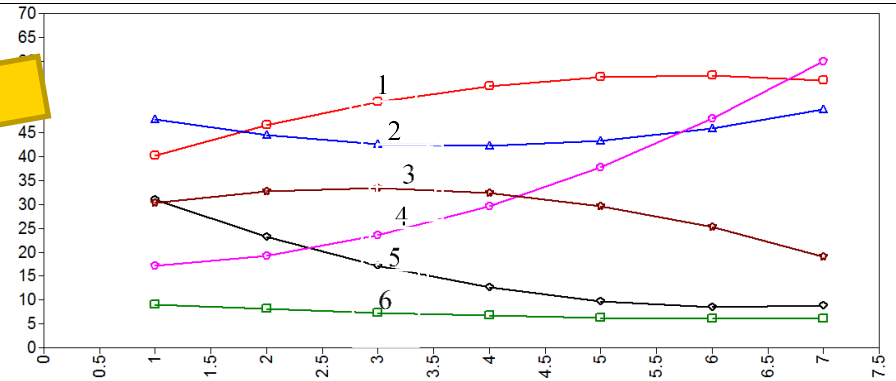
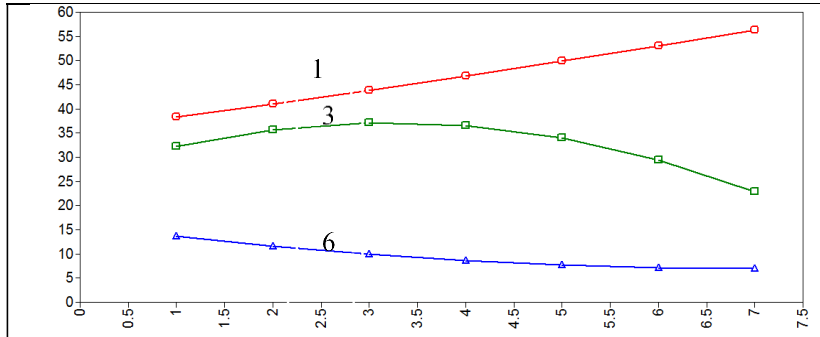
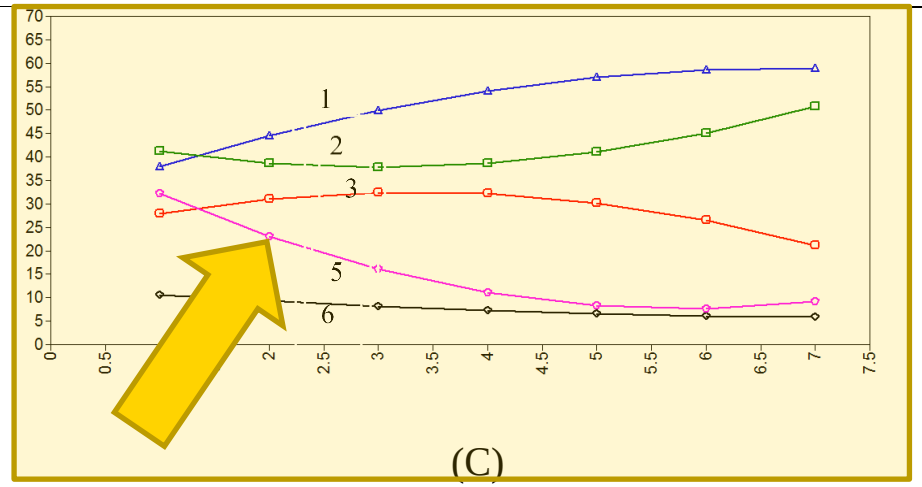


(D)

Empirical Example



(A)



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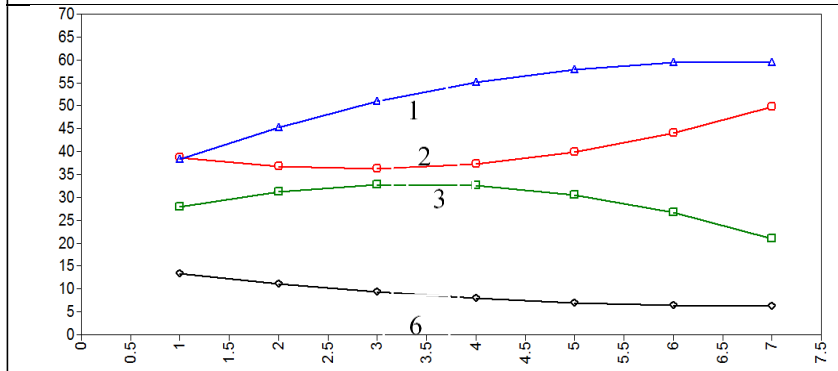

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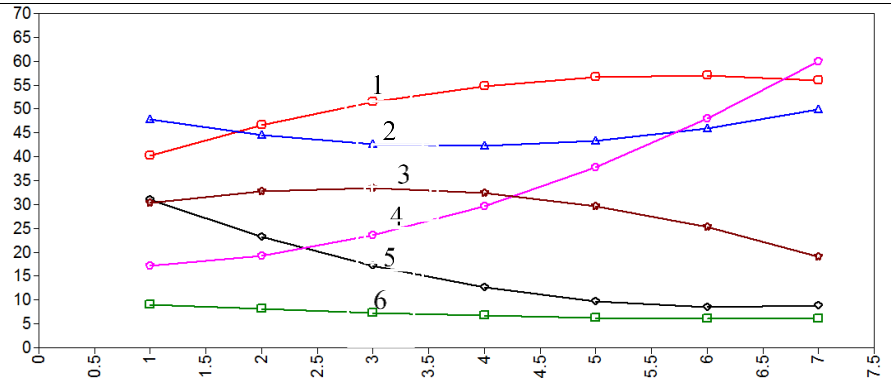
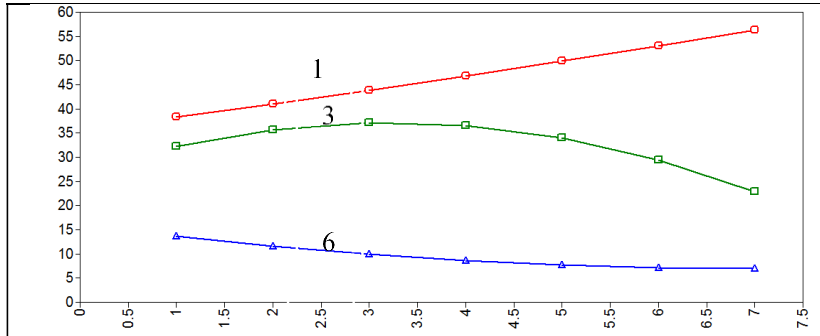
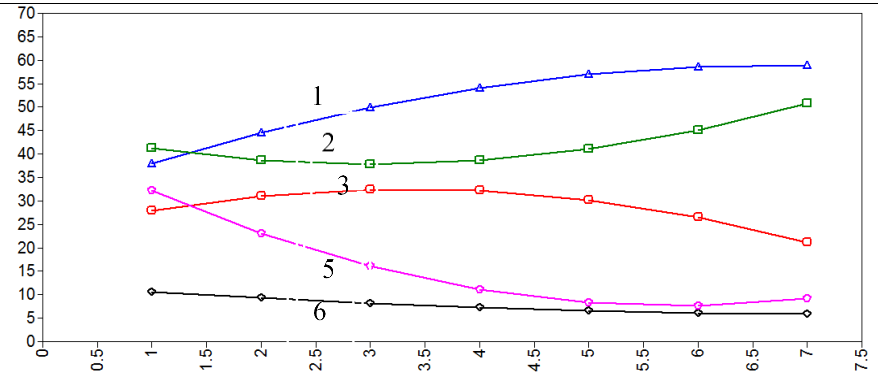


(D)

Empirical Example



(A)



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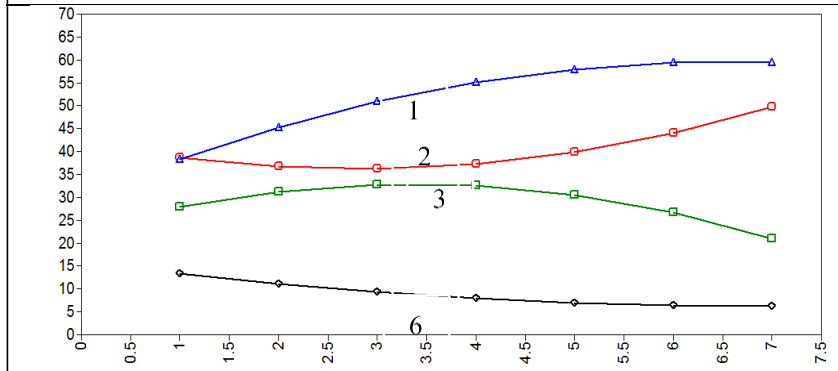

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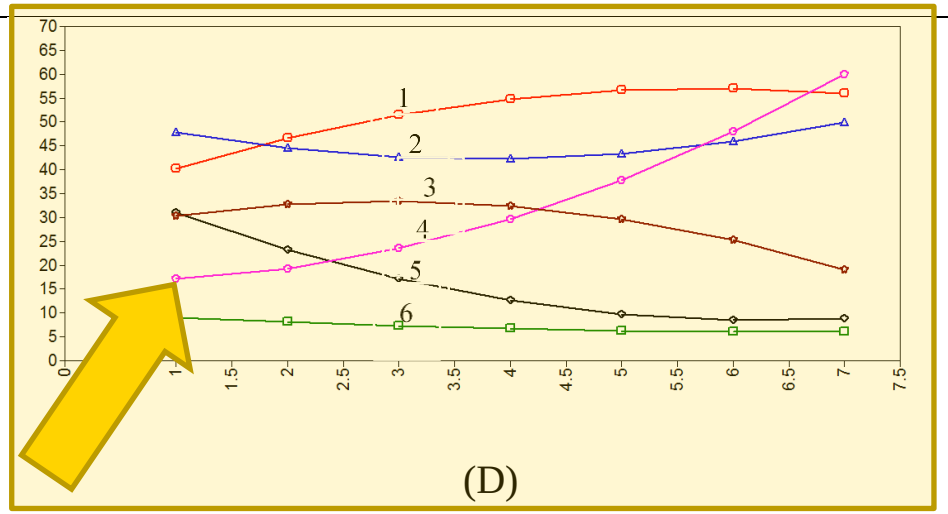


(D)

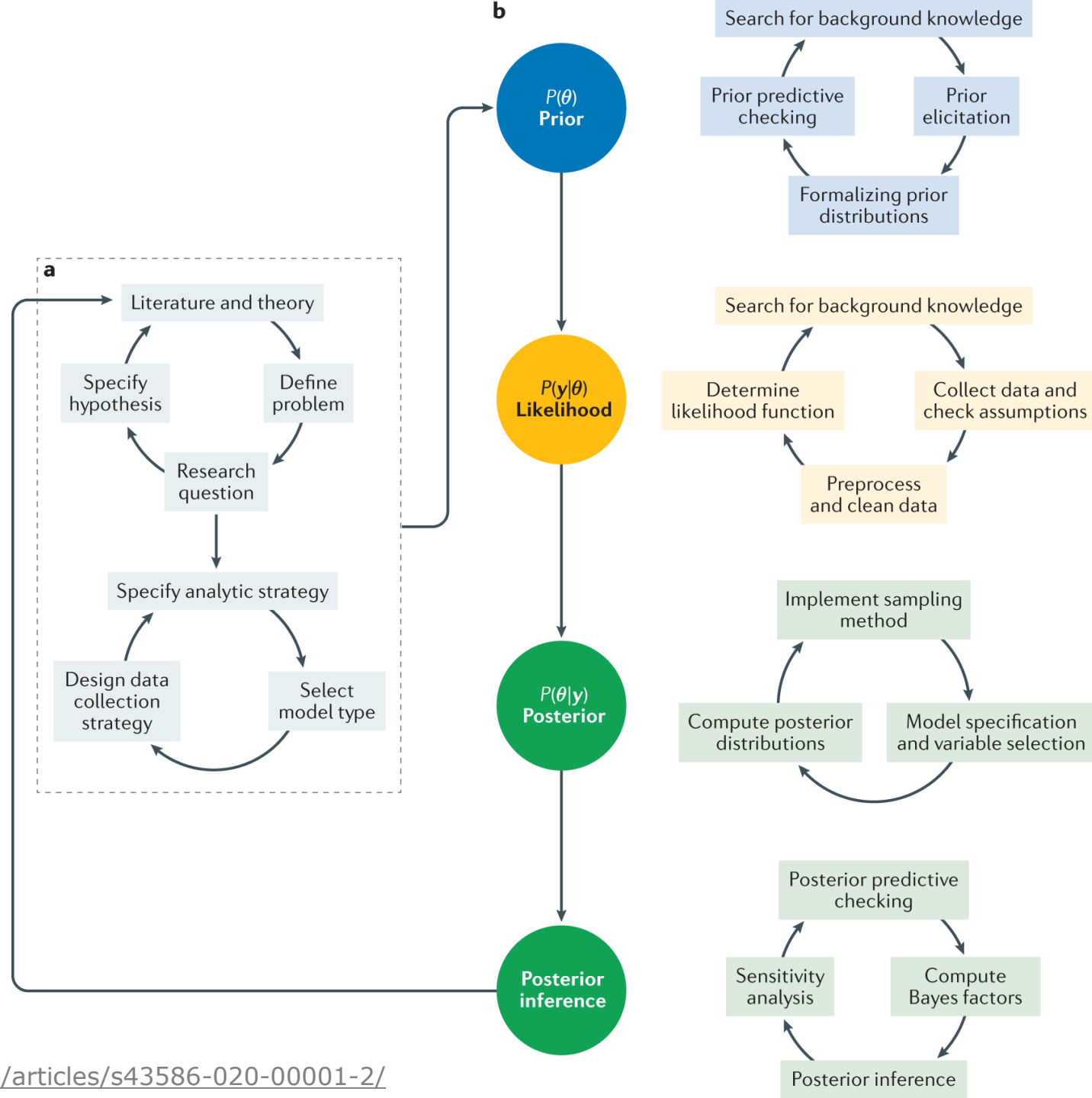
Issues

- Trajectories unrelated to any theory.
  - Note that these unrelated trajectories may just be pulling out outliers, for example, rather than substantive clusters.

- Latent trajectories that do exist in the population fail to show up in the data under consideration
  - Especially when a small cluster is expected

Integrating expert knowledge about traumatic stress into statistical models assessing individual change over time by using Bayesian statistics

Van de Schoot, Sijbrandij, Depaoli, Winter, Olff, van Loey (2018). Bayesian PTSD-Trajectory Analysis with Informed Priors Based on a Systematic Literature Search and Expert Elicitation. *Multivariate Behavioral Research – Tutorial Corner*
https://doi.org/10.1080/00273171.2017.1412293



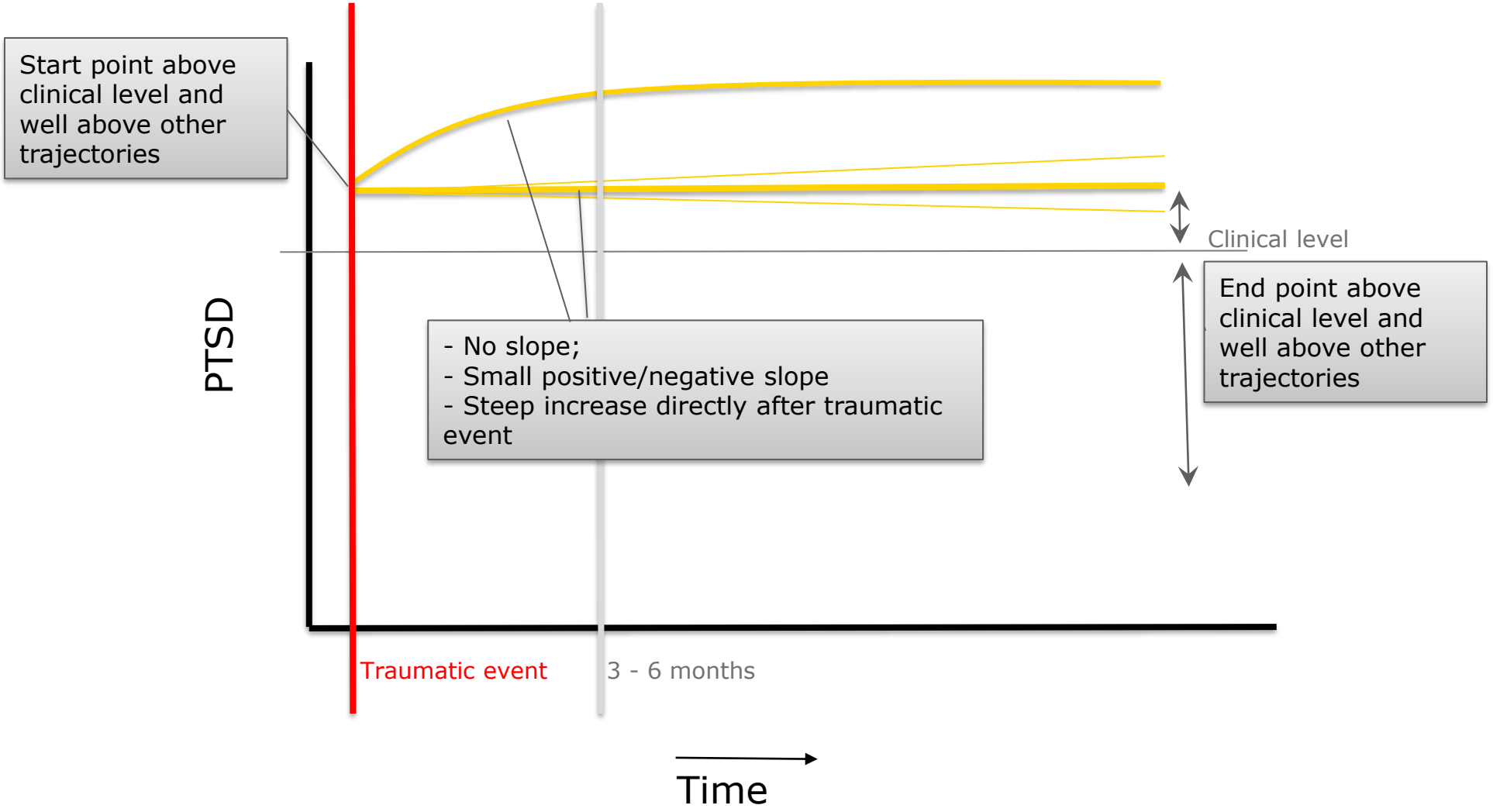
www.nature.com/articles/s43586-020-00001-2/

What do we know about the trajectories?

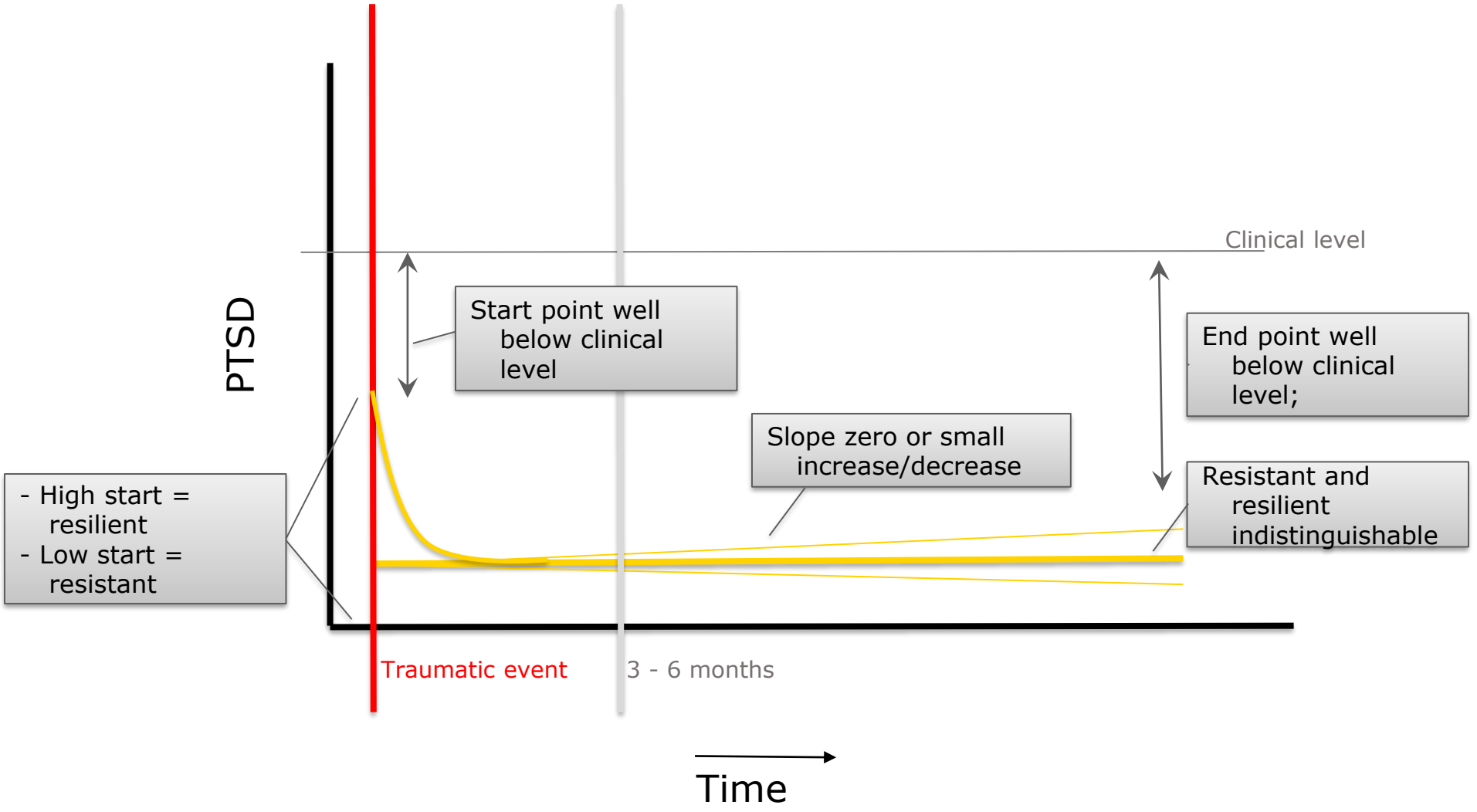
Based on experts discussions

(senior researchers working with PTSD data and applying LGMM/LCGA techniques to their data)

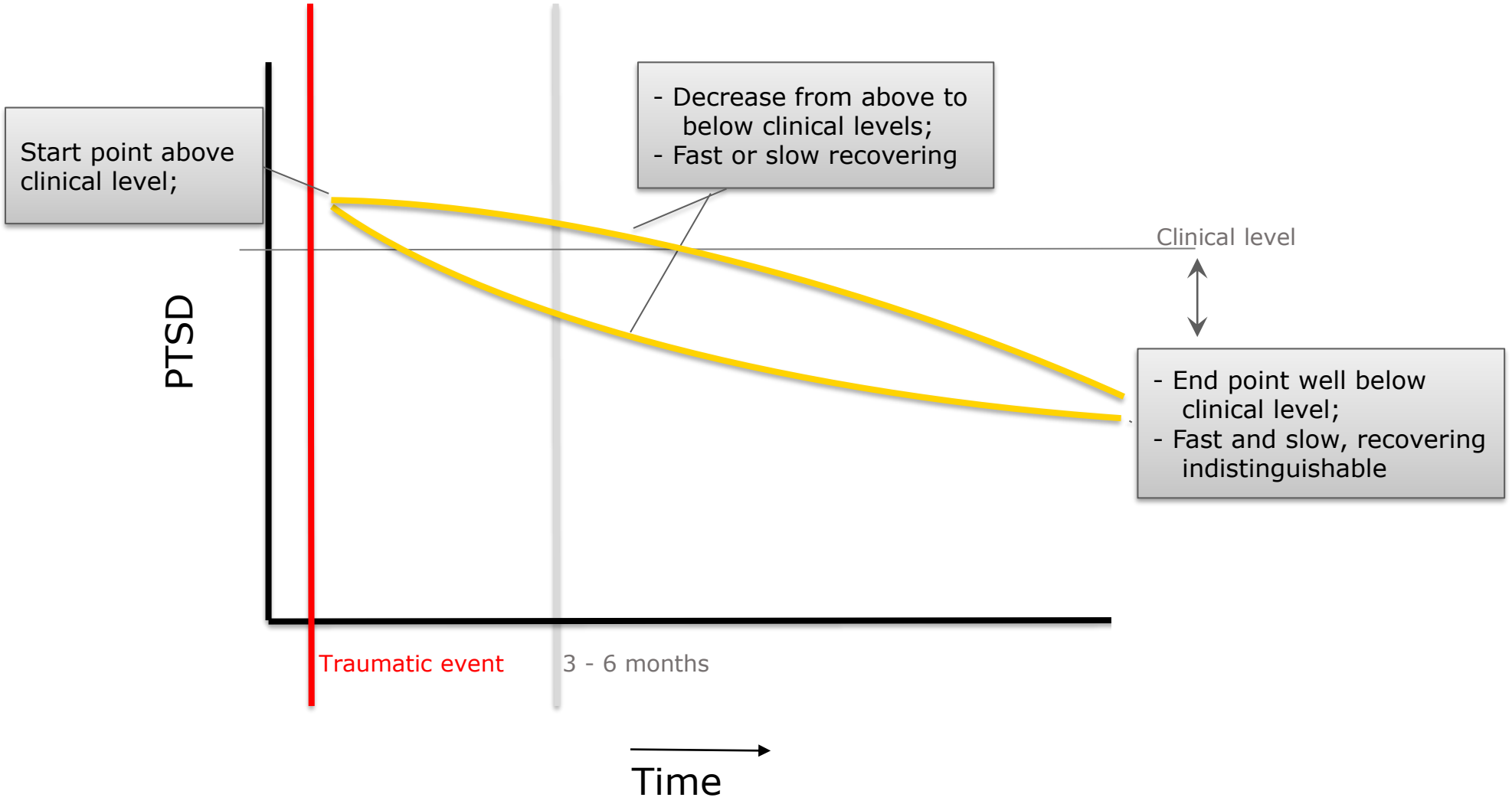
Chronic trajectory



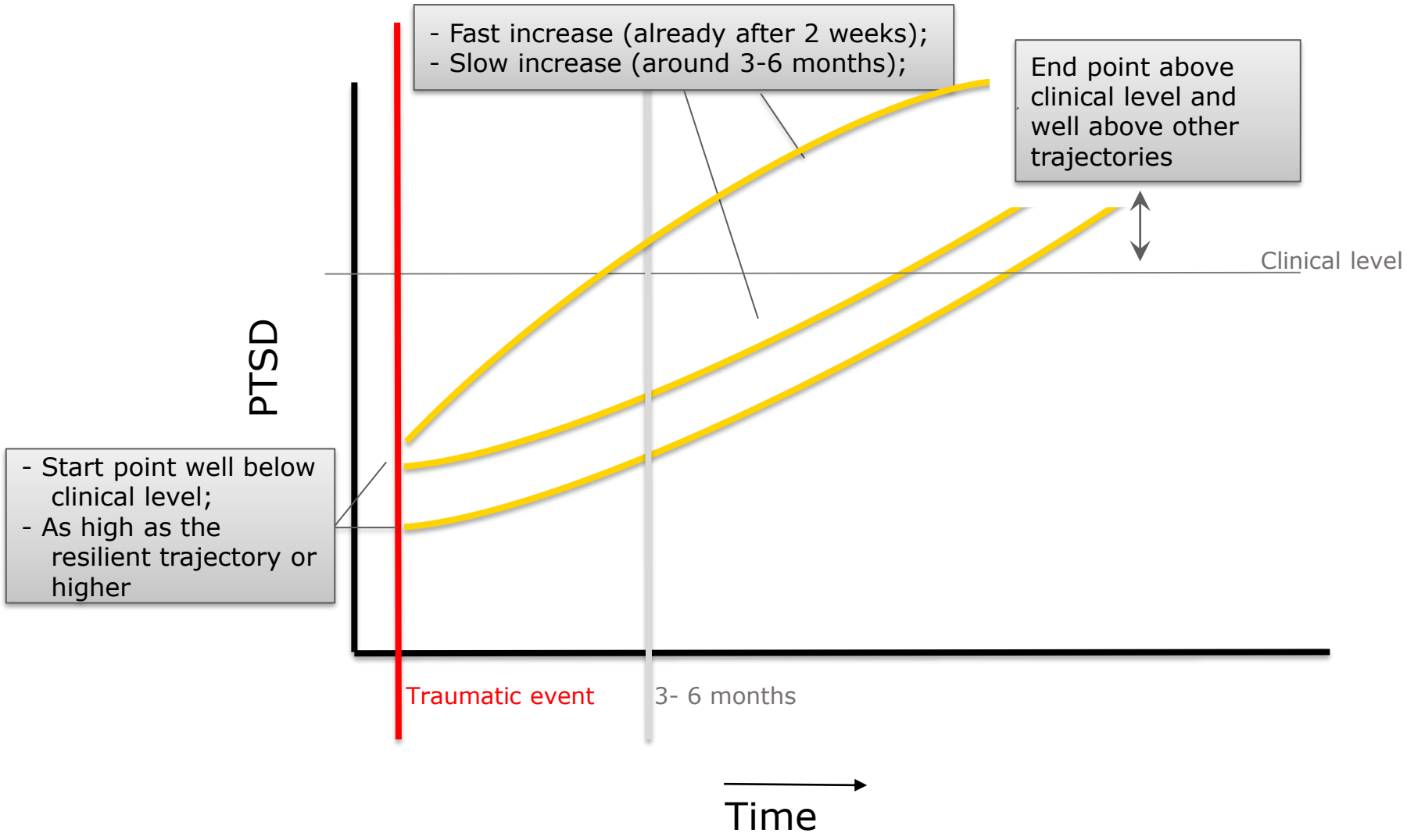
Resistant/resilient trajectory



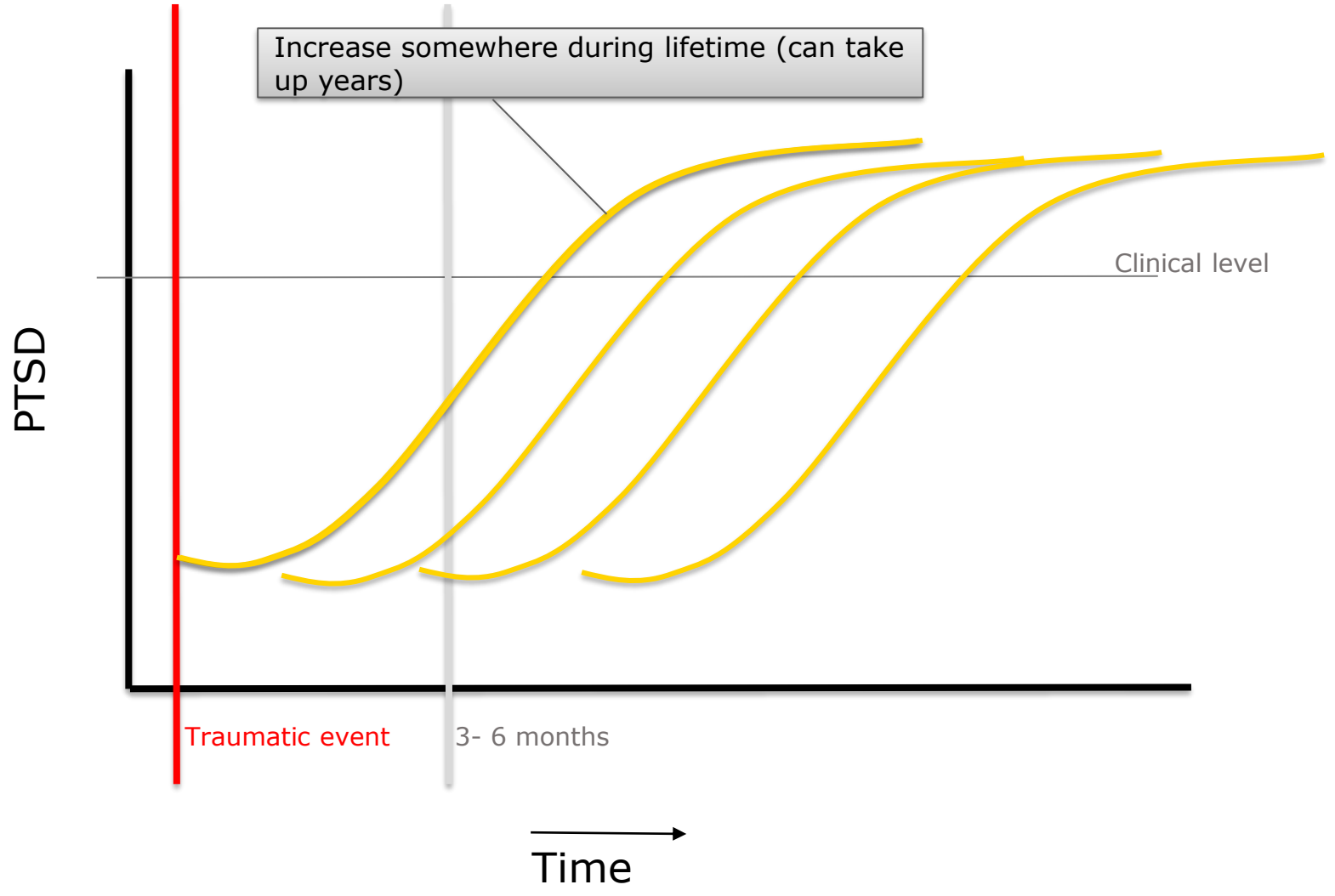
Recovering trajectory



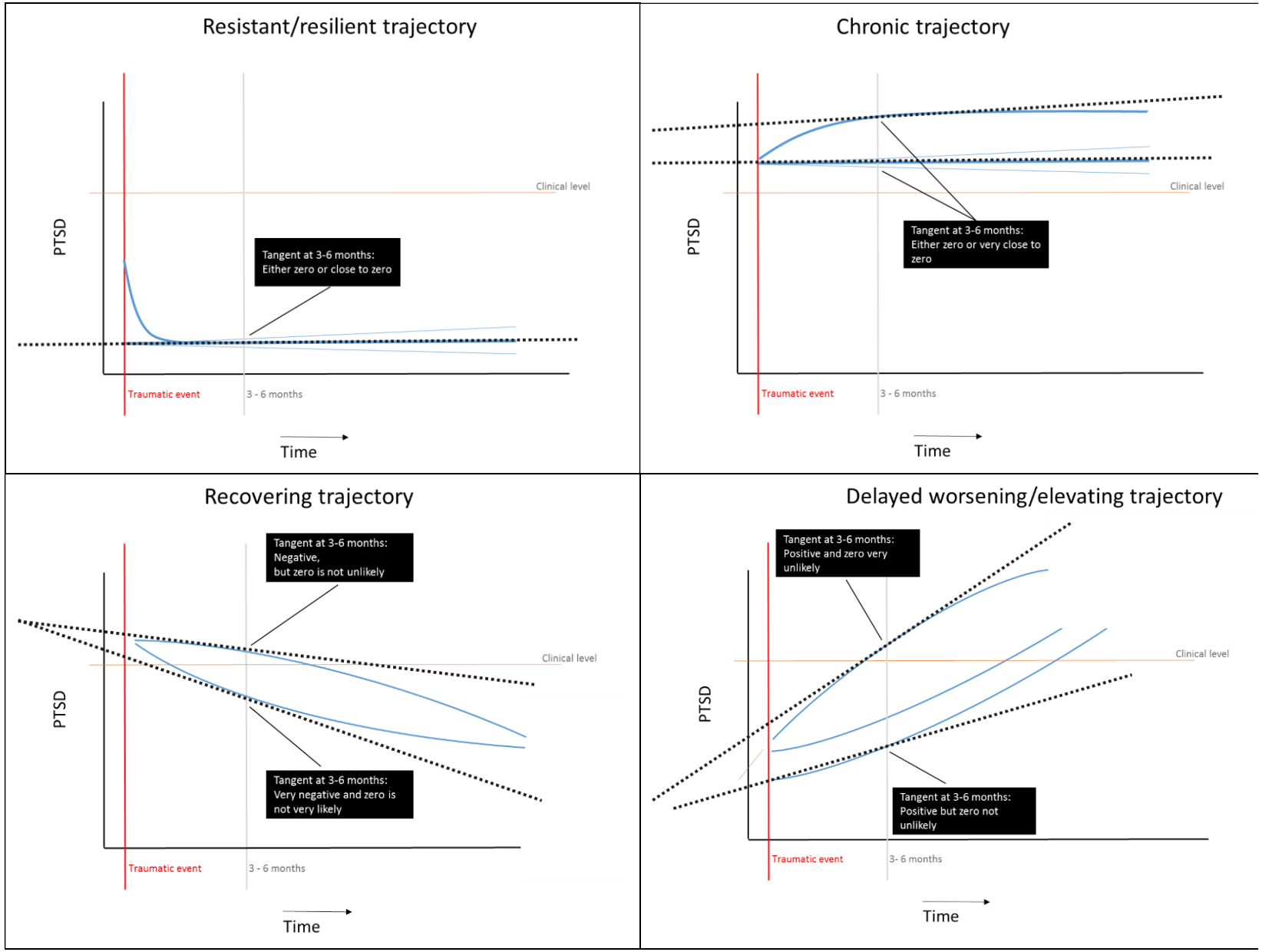
Delayed worsening/elevating trajectory



Delayed worsening/elevating trajectory



Information about the tangent lines (i.e., the slope parameter of the growth process) at three months after trauma, which is used for the prior specification.

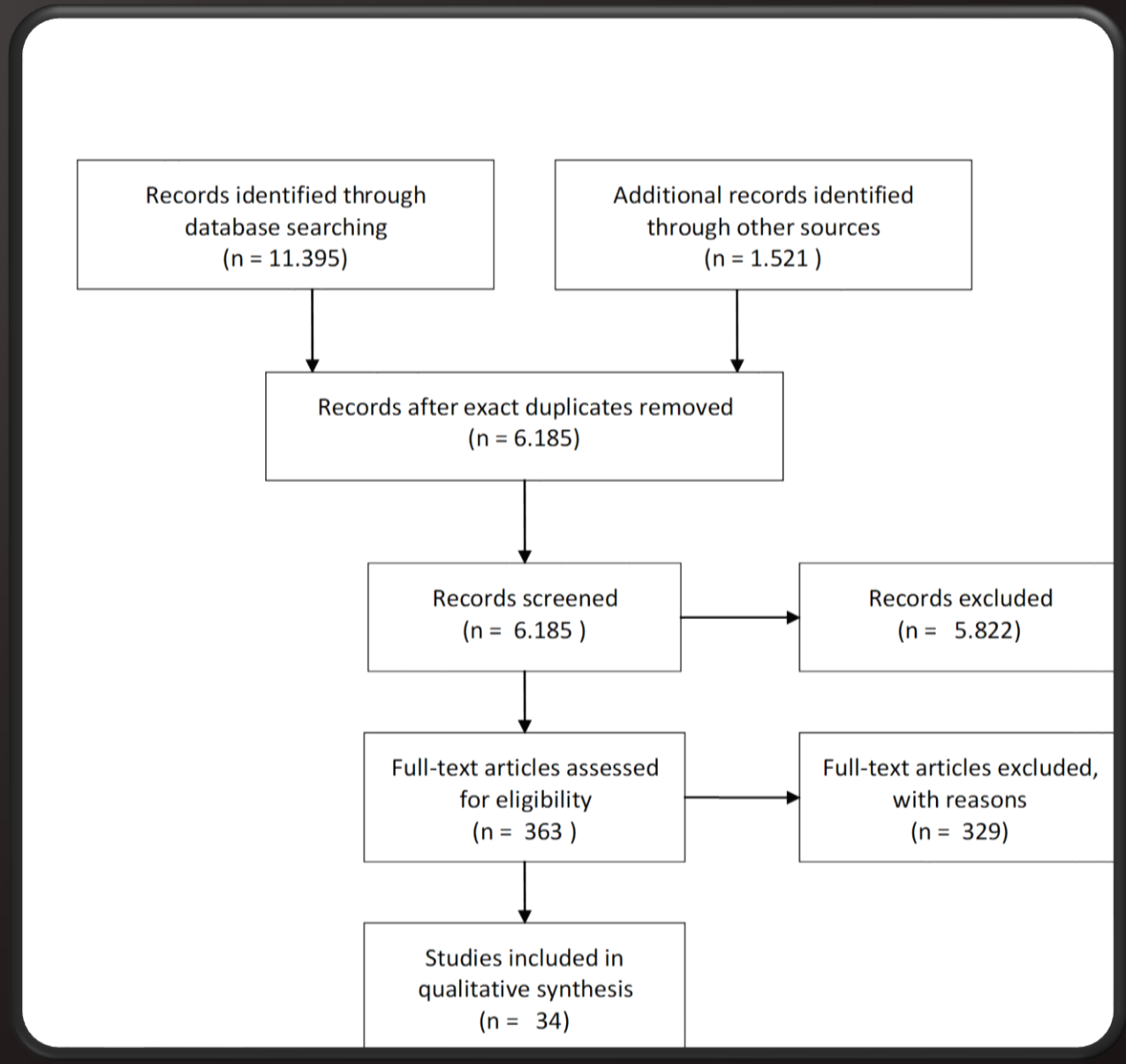


Search:

-Relevant databases (Pubmed, Embase, PsychInfo, Scopus)
-Relevant search terms (e.g., latent, trajectories, PTSD, trauma, stress)
-Title and abstract screening
-Study inclusion

Inclusion criteria:

-Longitudinal studies
-Traumatic event conform DSM-5
-Assessment of PTSD symptoms across at least 3 time points following trauma
- Using latent growth mixture modeling (LGMM) or Latent Class Growth Analysis (LCGA) to estimate trajectories

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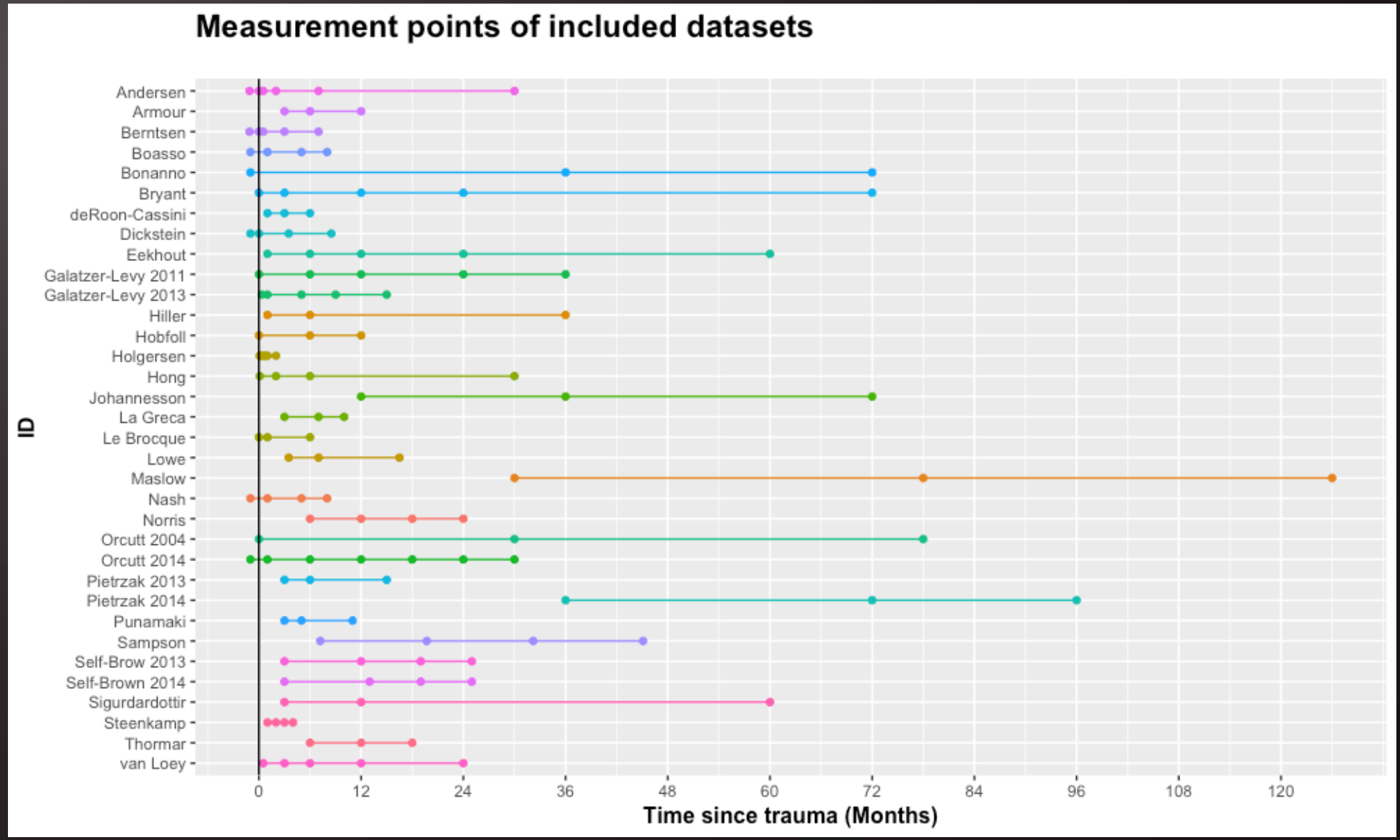
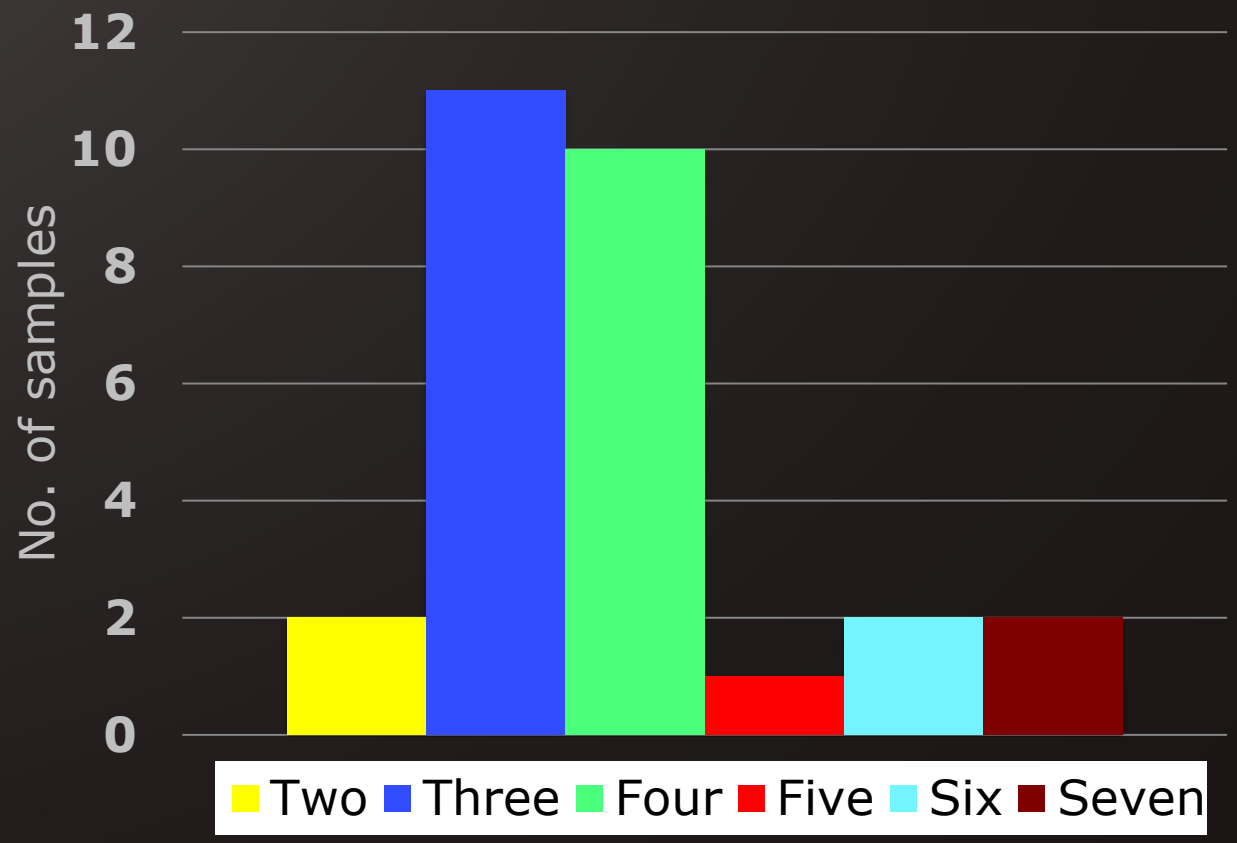


Van de Schoot, Sijbrandij, Depaoli, Winter, Olff, van Loey (in press). Bayesian PTSD-Trajectory Analysis with Informed Priors Based on a Systematic Literature Search and Expert Elicitation. *Multivariate Behavioral Research – Tutorial Corner*

Number of trajectories:



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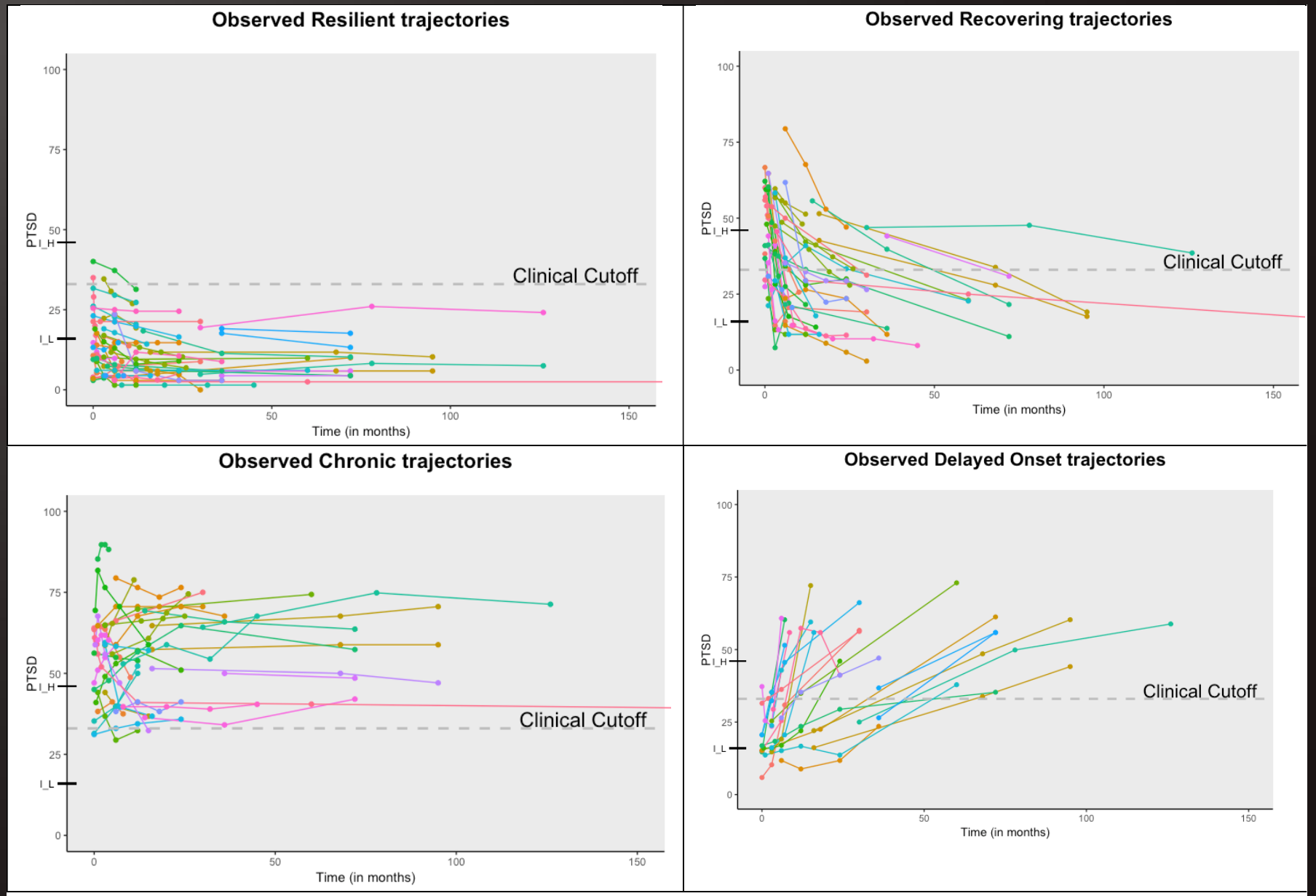

Observed Resilient trajectories
PTSD
I_H
I_L
Clinical Cutoff
Time (in months)
Observed Recovering trajectories
PTSD
I_H
I_L
Clinical Cutoff
Time (in months)
Observed Chronic trajectories
PTSD
I_H
I_L
Clinical Cutoff
Time (in months)
Observed Delayed Onset trajectories
PTSD
I_H
I_L
Clinical Cutoff
Time (in months)

Descriptive statistics of PTSD score at zero months for each trajectory.

Trajectory	Mean (SD)	Median	Min	Max
Resilient (n = 14)	17.27 (12.67)	13.98	2.94	40.06
Delayed/ Worsening/ Elevating (n = 9)	19.76 (9.43)	16.85	5.88	37.25
Recovering (n = 9)	46.42 (14.84)	40.94	27.45	66.67
Chronic (n = 7)	48.94 (12.94)	47.06	31.13	64.00

Van de Schoot, Sijbrandij, Depaoli, Winter, Olff, van Loey (in press). Bayesian PTSD-Trajectory Analysis with Informed Priors Based on a Systematic Literature Search and Expert Elicitation. *Multivariate Behavioral Research – Tutorial Corner*

Summary of the slope parameters found in 34 papers reported on LGMM.

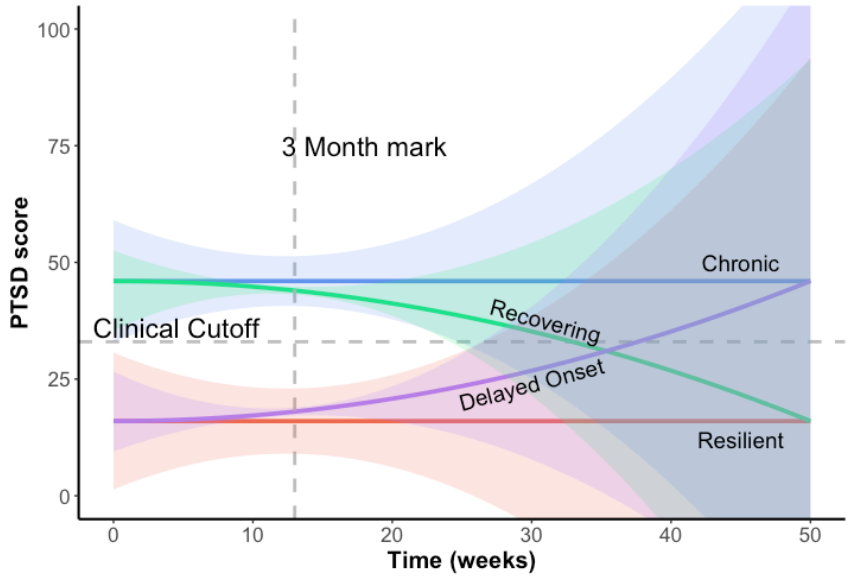
	Nr. of slope parameters not significant	Nr. of slope parameters significant	Direction slope if significant	If quadratic, U-shape or ∩-shape if significant
Resilient	8	15	10x negative 5x positive	3 x U 5 x ∩
Chronic	8	10	2x negative 8x positive	1 x U 5 x ∩
Recovering	3	20	2 x pos 18 x negative	5 x U 2 x ∩
Delayed/ Worsening/ Elevating	1	8	6 x positive 2 x negative	3 x U 4 x ∩

Therefore
2 parameterizations

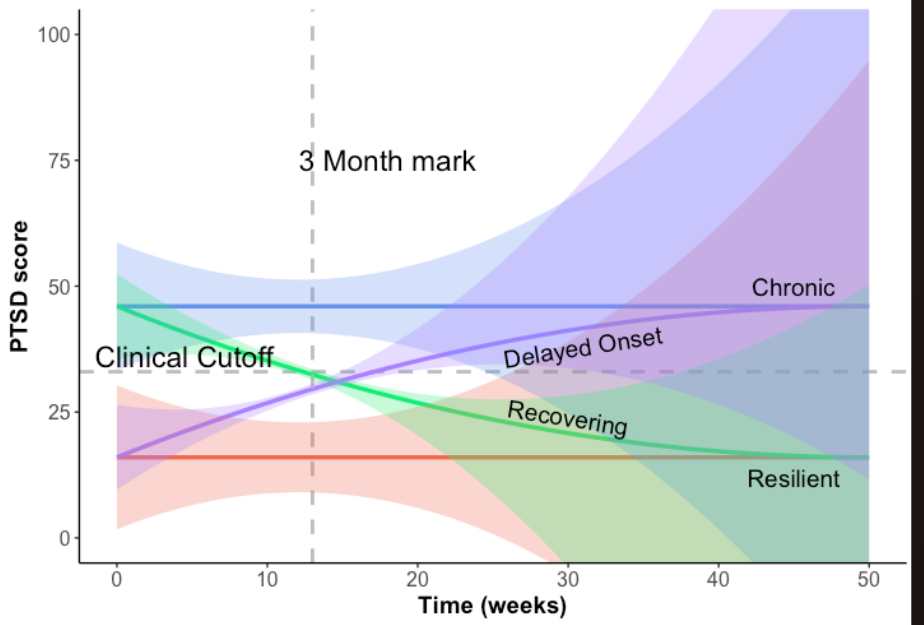
Van de Schoot, Sijbrandij, Depaoli, Winter, Olff, van Loey (in press). Bayesian PTSD-Trajectory Analysis with Informed Priors Based on a Systematic Literature Search and Expert Elicitation. *Multivariate Behavioral Research – Tutorial Corner*

The bold lines represent the trend lines of the growth in PTSD symptoms based on the background information and for two different parameterizations for the recovery and delayed-onset trajectory (i.e., U-shaped or ∩-shaped). The shaded areas represent the uncertainty around the average trend lines as specified in the prior distributions.

Parameterization 1:



Parameterization 2:



Van de Schoot, Sijbrandij, Depaoli, Winter, Olff, van Loey (in press). Bayesian PTSD-Trajectory Analysis with Informed Priors Based on a Systematic Literature Search and Expert Elicitation. *Multivariate Behavioral Research – Tutorial Corner*

Empirical example data

- **Patients with burn injuries admitted to a burn center**
- **February 1997 and February**
- **Netherlands and Belgium**
- **N=301**
- ***M* = 38.5 years, *SD* = 13.5**
- **informed consent**
- **assessed twice during hospitalization and subsequently, every 8 weeks until 12 months after the burn event**
- **Dutch version of IES**

Van Loey, Faber, Maas (2003). Predictors of chronic post-traumatic stress symptoms following burn injury: Results of a longitudinal study. *Journal of Traumatic Stress*

Priors

The prior for the class proportion parameters is the Dirichlet distribution

with the hyperparameters $\delta_1, \delta_2, \ldots, \delta_K$

These hyperparameters represent the proportion of cases assumed to be in the *K* latent classes.

Priors

Bonanno (2004) provides approximations of the proportion of each of the four trajectories based on a number of empirical studies:

1. 75% - resilient individuals
2. 11.25% - recovering PTSD
3. 7.5% - chronic PTSD
4. 6.25% - delayed onset PTSD

With a total sample size of 301, the numerical hyperparameters for the Dirichelt prior are:

MODEL PRIORS:

d1~D(226,19);

d2~D(34,19);

d3~D(22,19);

Resistant/resilient trajectory:

```
I ~ N(16, 106.9156)
S ~ N(0, 0.00809)
Q ~ N(0, 1)
```

Recovering trajectory:

```
I ~ N(43.972, 1.5129)
S ~ N(-.156, 0.00809)
Q ~ N(-.012, 1)
```

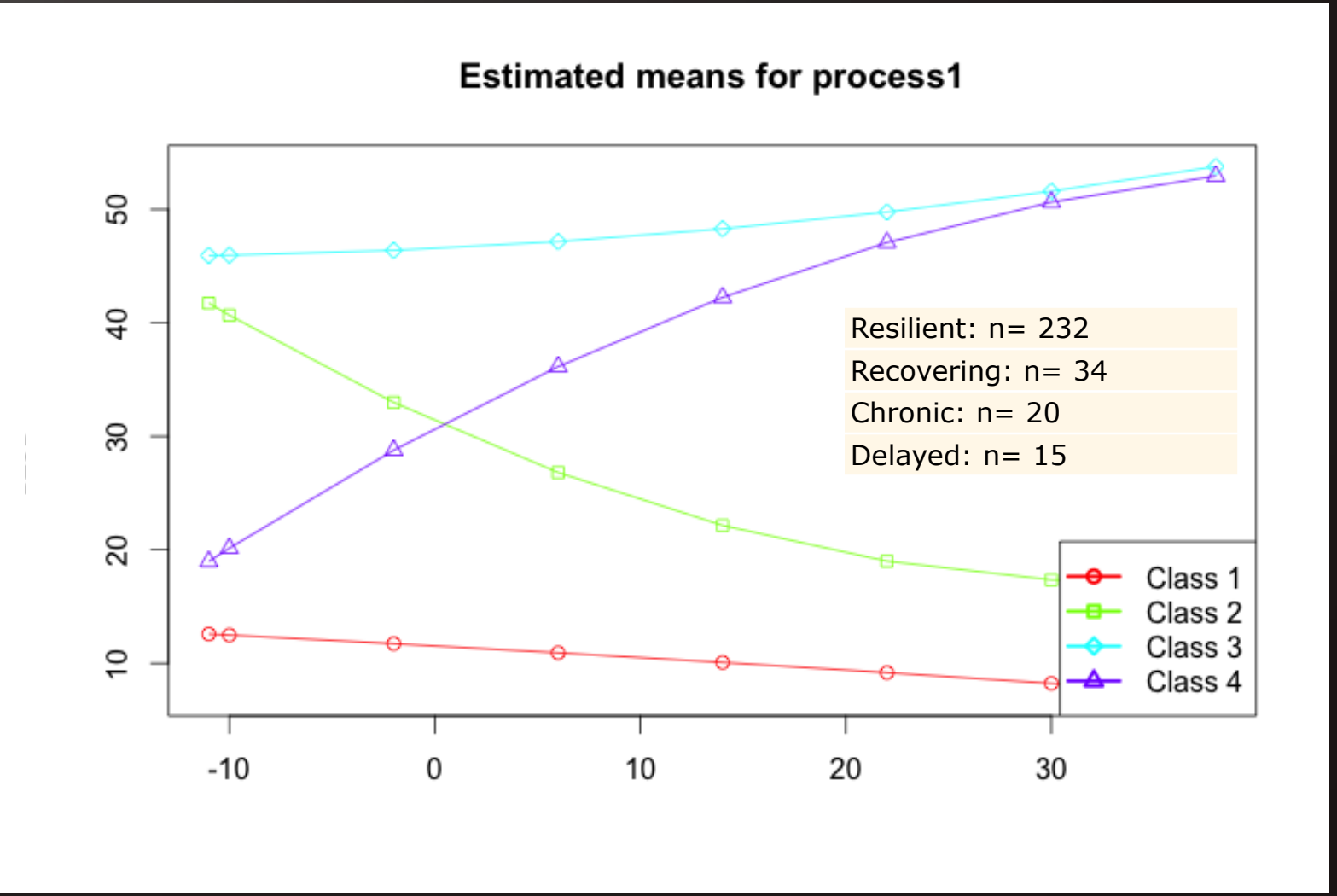
Chronic trajectory:

```
I ~ N(46, 62.41)
S ~ N(0, 0.00809)
Q ~ N(0, 1)
```

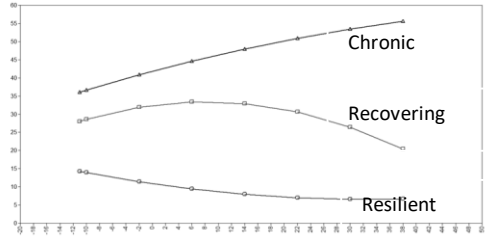
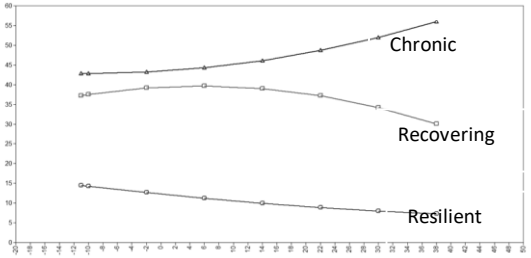
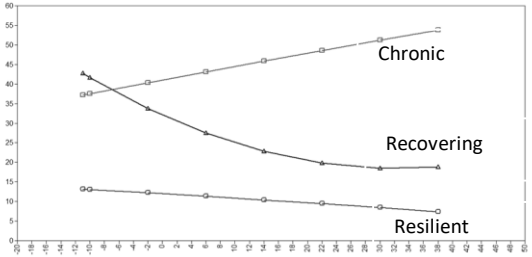
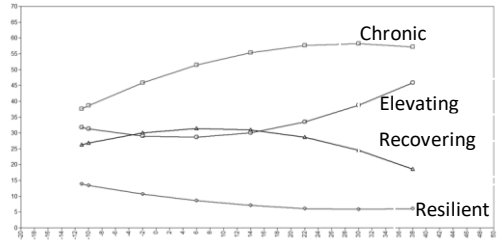
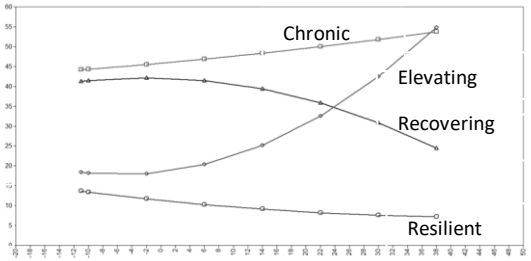
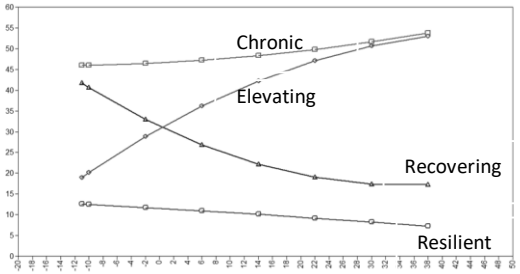
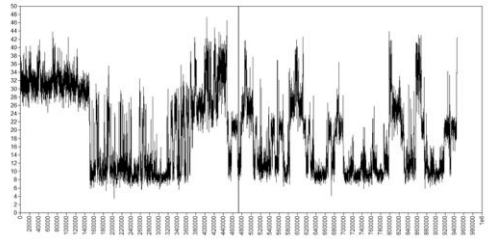
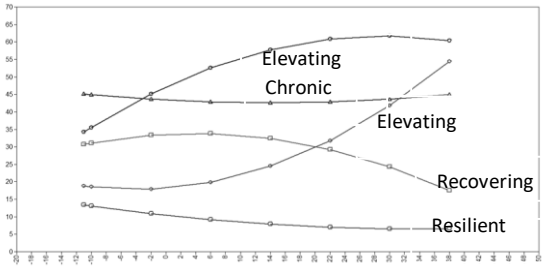
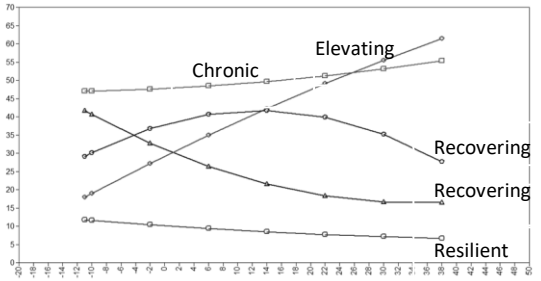
Delayed onset trajectory:

```
I ~ N(18.028, 1.5129)
S ~ N(.156, 0.00809)
Q ~ N(.012, 1)
```

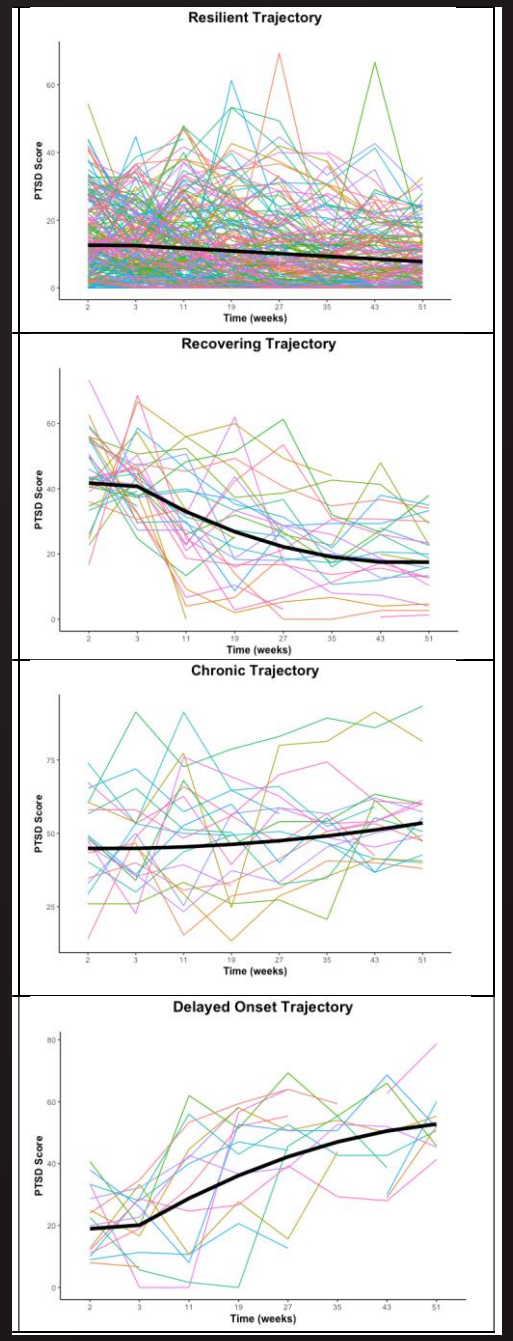
Estimated trajectories



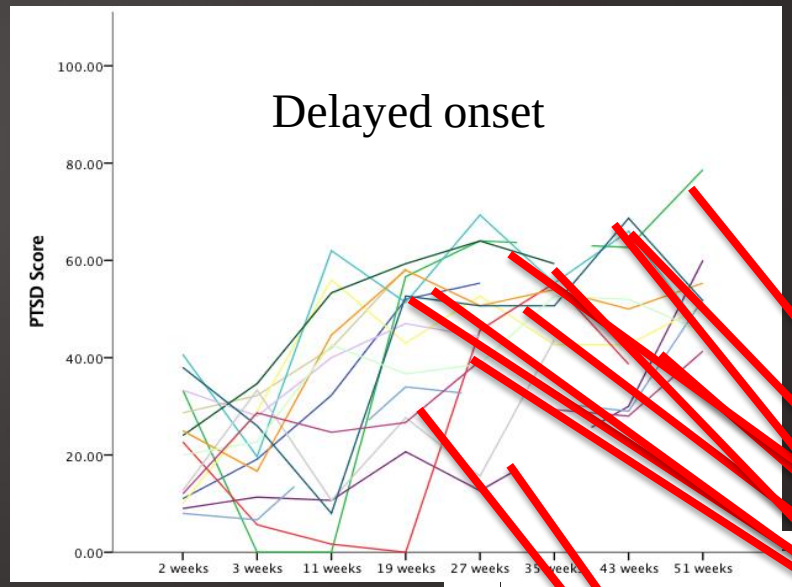
Graphical representation of the posterior results for the nine different models we fitted on the data.

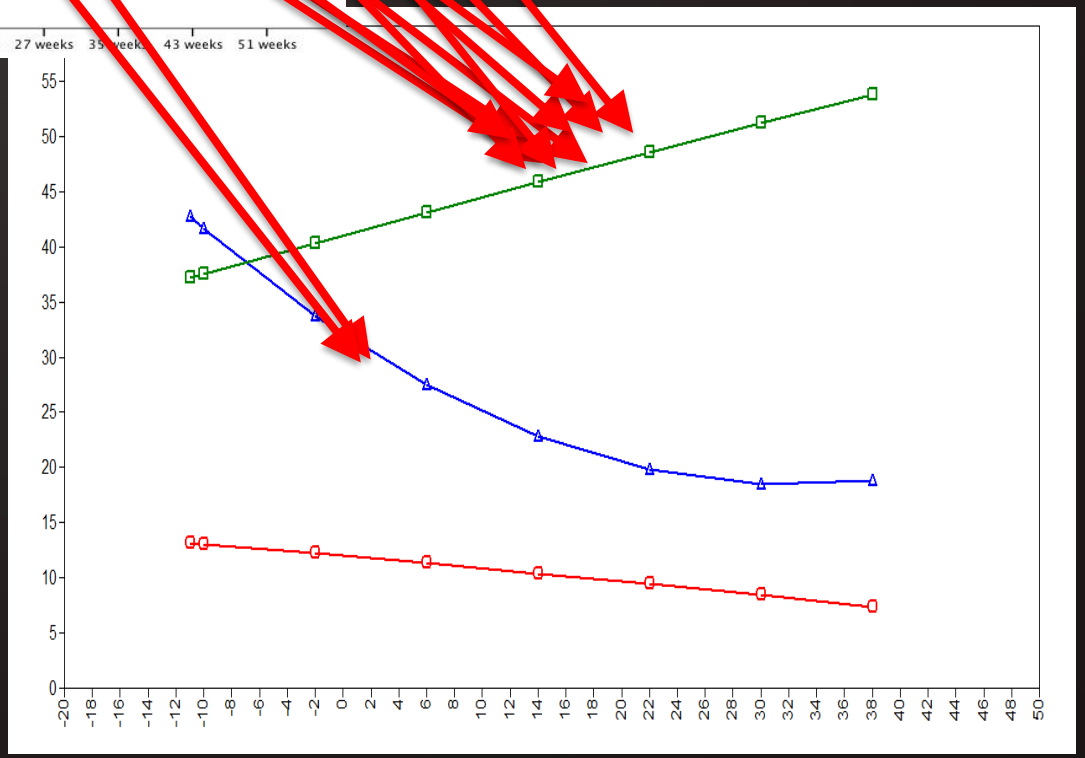
	Default priors	Parameterization 1	Parameterization 2
3 classes			
4 classes			
5 classes	 Serious label switching issues even after a million iterations, see for example the trace plot of the mean intercept of the fifth class.		

Observed individual trajectories per type of trajectory and for both parameterizations. To classify individuals in the four trajectories, we saved the plausible values with 500 imputed datasets for class membership. The resulting file was imported into SPSS and an average class membership was estimated. For example, if an individual showed the pattern 0/0/0/100 (where 100% of the imputations assigned this person to Class 4), then this person was allocated to the delayed onset trajectory (i.e., the fourth class). If the pattern was 0.11/0.27/0.3/0.59, then the person was allocated to the delayed onset trajectory in 59% of the imputations. Most likely class membership was based on highest count.

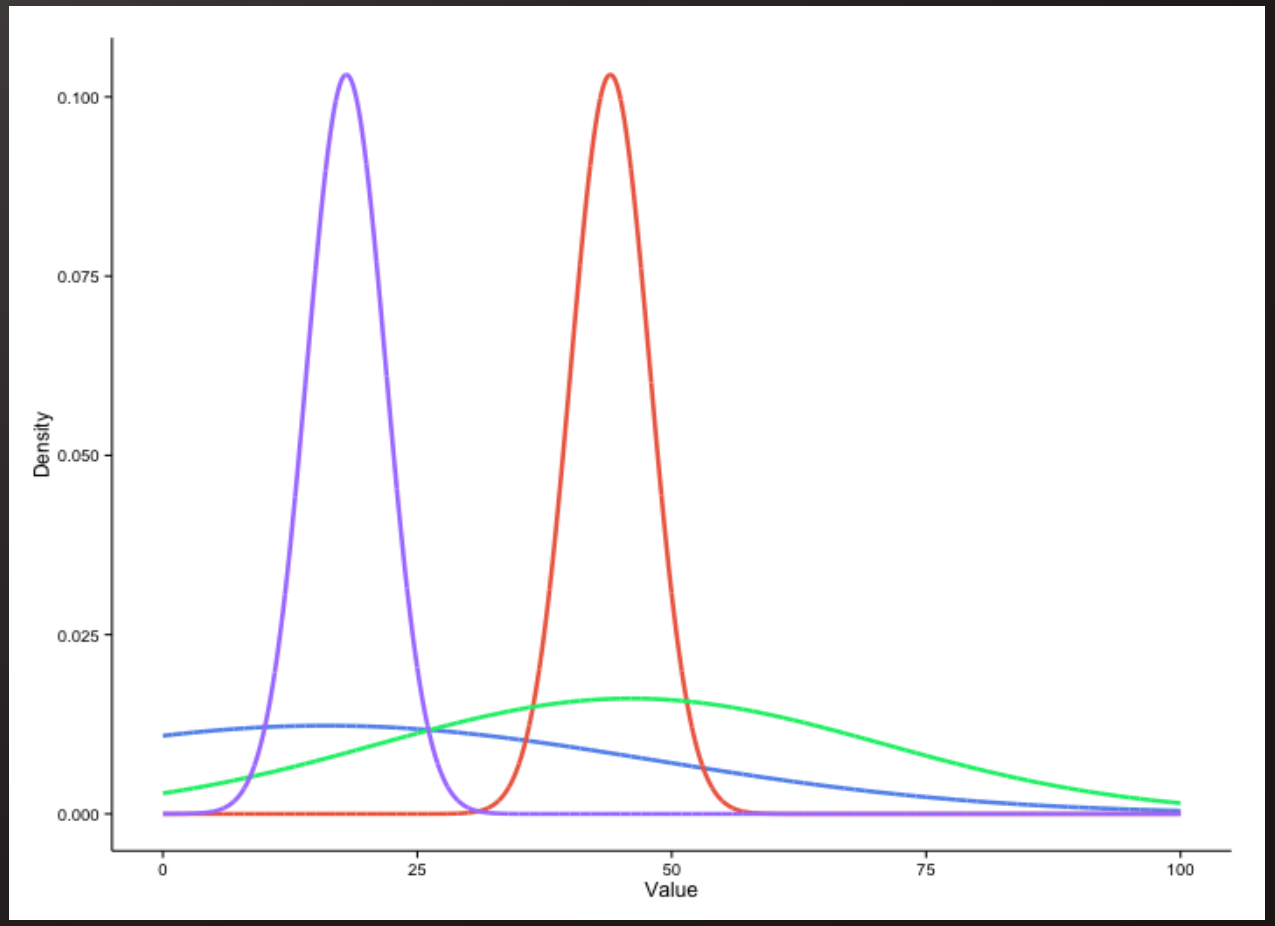


Results sensitivity Analysis

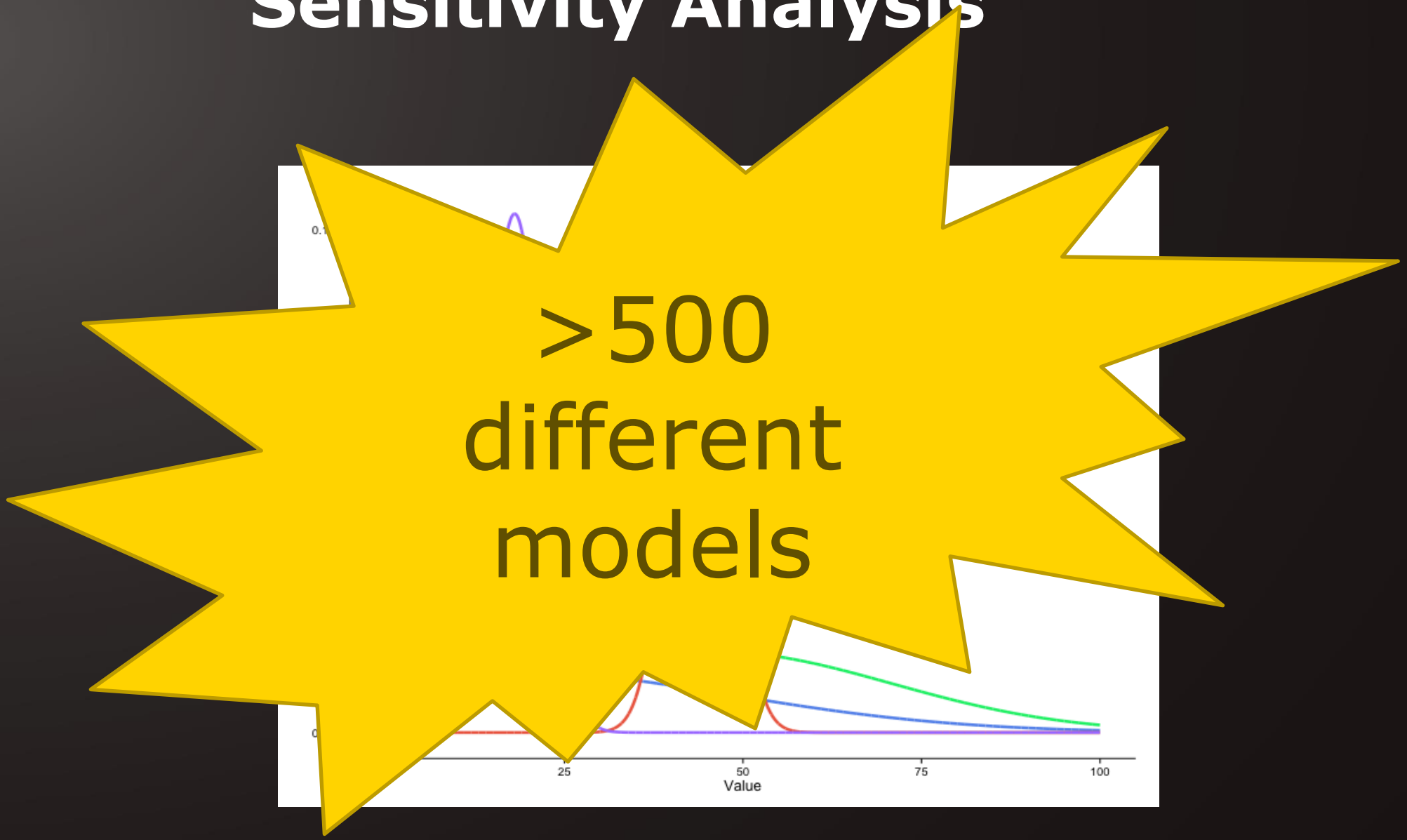




Sensitivity Analysis



Sensitivity Analysis



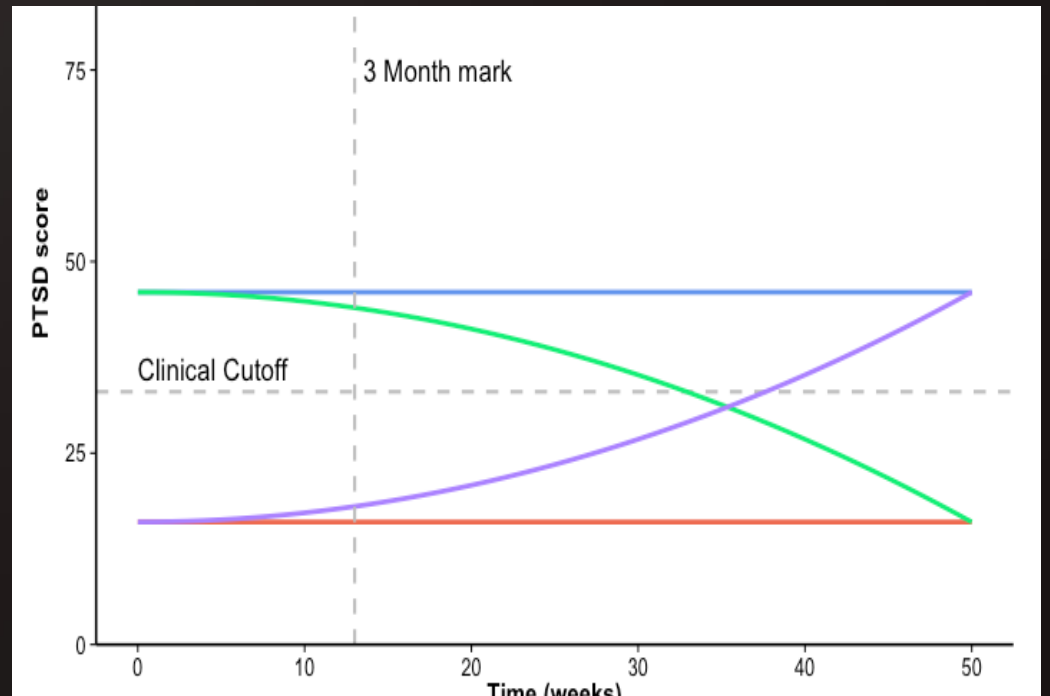
Results sensitivity Analysis

Results sensitivity Analysis

- Quite some wiggle room, bias <5%
- Results are not sensitive for slightly different values (i.e, it does not matter if you specify the intercept as 37, 48 or 51)

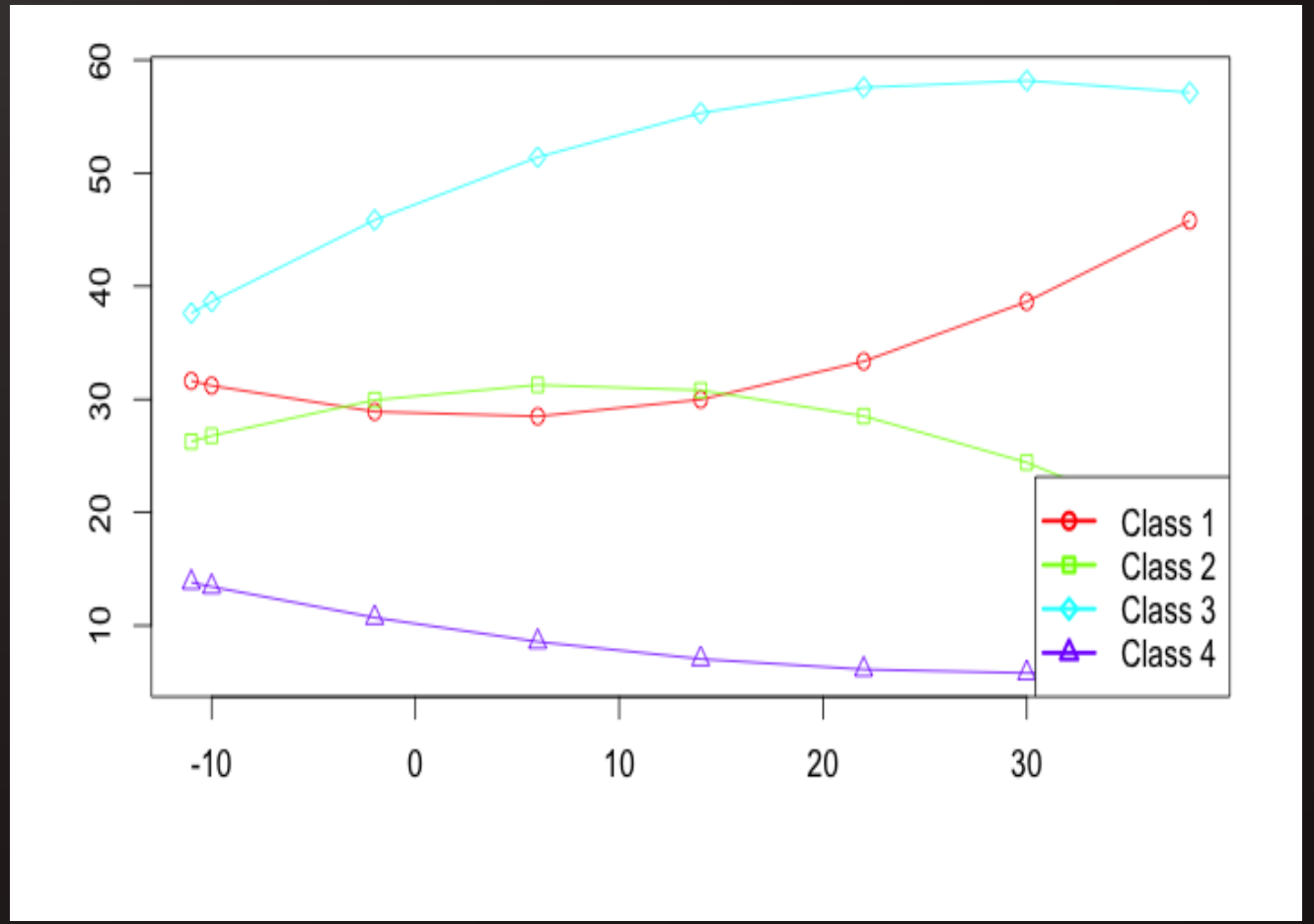
Results sensitivity Analysis

- Shape of the trajectory does matter
  - Convergence issues
  - Substantially different solutions



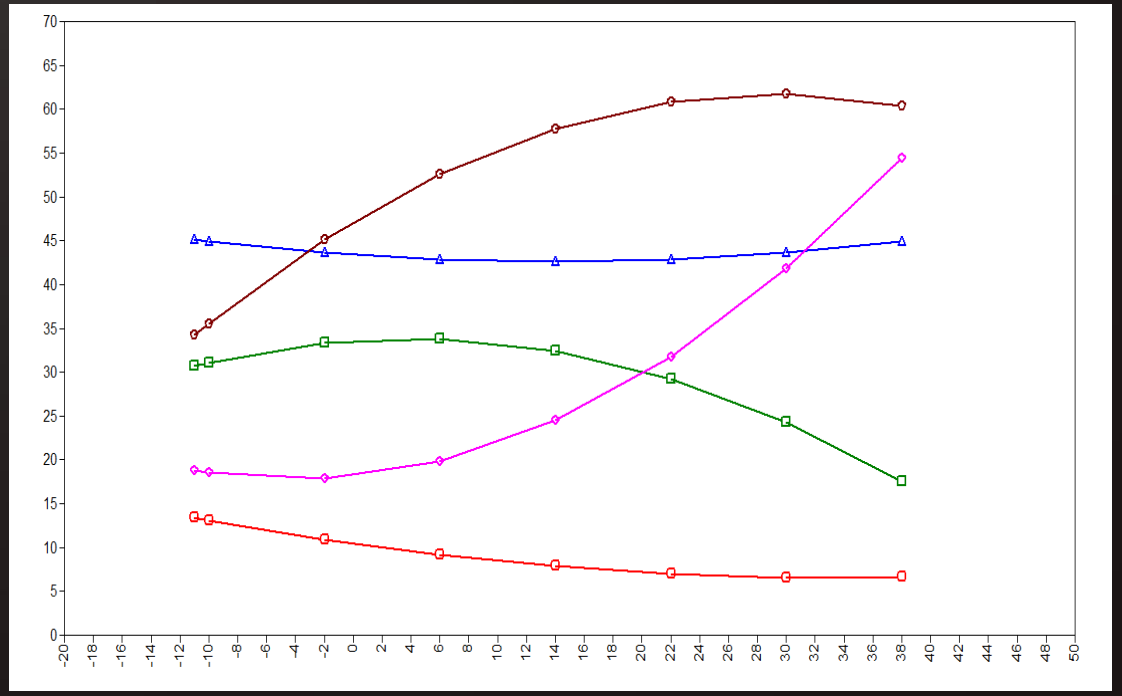
Results sensitivity Analysis

Informativeness of the prior does matter



K+1 trajectories

The estimated means for the Bayesian 5-class model where priors for the 5th class are uninformative.



Main Message

Specific expectations produces more useful results than using an exploratory strategy

We propose:

- Use background knowledge about the shape of the trajectories

- Translate this knowledge into statistical prior distributions

- Estimate informed-LGMM instead of an exploratory LGMM model